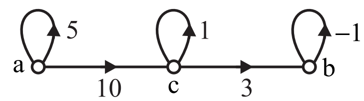
Classroom Test : 2016 - 17

Control Systems (EC /EE/EEE/ IN)

Duration : 90 Minutes **Maximum Marks : 50**

Q.1 to Q.10 carry one mark each

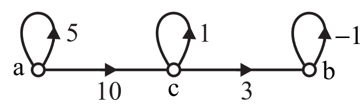
Q.1 Consider the SFG shown in below figure.



Then $\frac{b}{a}$ is _____.

Ans. **15 to 15 (15)**

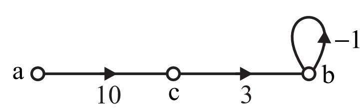
Sol.



Since, self loop at input node is invalid, so loop at node 'a' is invalid.

Since, self loop of value 1 is invalid, so loop at node 'c' is invalid.

Hence, the SFG becomes



Using mason's gain formula

Forward path gain $P_1 = 3 \times 10 = 30$

A loop $L_1 = -1$

Associated path factor with path P_1

$$\Delta_1 = 1 - (0) = 1$$

Total path factor

$$\Delta = 1 - (-1) = 2$$

Hence, By mason's gain formula

$$\frac{b}{a} = \frac{\Sigma P_k \Delta_k}{\Delta} = \frac{P_1 \Delta_1}{\Delta}$$

$$\frac{b}{a} = \frac{30 \times 1}{2} = 15$$ **Ans.**

Q.2 Given a function $e_{ss} = \frac{\alpha}{k^2}$. The sensitivity of e_{ss} with respect to k at $\alpha = 1$ and $k = 1$ is ____.

Ans. – 2 to – 2 (– 2)

Sol. Given :

$$e_{ss} = \frac{\alpha}{k^2} \quad \text{...(i)}$$

To find sensitivity of e_{ss} with respect to k i.e. $S_k^{e_{ss}}$ will be defined as

We know, sensitivity
$$S_k^{e_{ss}} = \frac{\frac{\partial e_{ss}}{e_{ss}}}{\frac{\partial k}{k}} = \frac{\partial e_{ss}}{\partial k} \cdot \frac{k}{e_{ss}} \quad \text{...(ii)}$$

Differentiating e_{ss} with respect to k

$$\Rightarrow \quad \frac{\partial e_{ss}}{\partial k} = \frac{-2\alpha}{k^3}$$

From (ii)
$$S_k^{e_{ss}} = \frac{-2\alpha}{k^3} \cdot \frac{k}{e_{ss}} = \frac{-2\alpha}{k^2 e_{ss}}$$

From (i)
$$S_k^{e_{ss}} = \frac{-2\alpha}{k^2 \cdot \frac{\alpha}{k^2}} = -2$$

Note : Sensitivity of e_{ss} is a constant independent of value α and k.

Q.3 The open loop transfer function of a unity feedback control system is given below. The value of K such that the system is oscillatory is _____.

$$G(s) = \frac{K}{s(s^3 + 3s^2 + 11s + 27)}$$

Ans. 18 to 18 (18)

Sol. Given :

$$G(s) = \frac{K}{s(s^3 + 3s^2 + 11s + 27)}$$

The characteristic equation of the system is $1 + G(s) = 0$

$$\Rightarrow \quad s^4 + 3s^3 + 11s^2 + 27s + K = 0$$

Making Routh-Hurwitz table

s^4	1	11	K
s^3	3	27	
s^2	2	K	
s^1	$\frac{54-3K}{2}$	0	
s^0	K		

Since, for system to be oscillatory, we need R.O.Z. formation at odd power of s

Here R.O.Z can form at s^1

Hence, $\frac{54-3K}{2} = 0$

Hence, $K = 18$ **Ans.**

As for $K = 18$, R.O.Z formation takes place and system is oscillatory.

Q.4 The closed loop transfer function of a system is given as

$$T(s)=\frac{10}{s^2+8s+8}$$

The settling time to an input of $5u(t)$ will be ______.

Ans. **3.2 to 3.6 (3.41)**

Sol.
$$T(s)=\frac{10}{s^2+8s+8}$$

By comparison with standard characteristic equation

$$s^2+2\xi\omega_n s+\omega_n^2=0$$

$\Rightarrow$ $\omega_n^2=8$

$\Rightarrow$ $\omega_n=2\sqrt{2}$

$\Rightarrow$ $2\xi\omega_n=8$

$\Rightarrow$ $\xi=\frac{8}{2\sqrt{2}.2}=\sqrt{2}$

$\Rightarrow$ $\xi=\sqrt{2}>1$

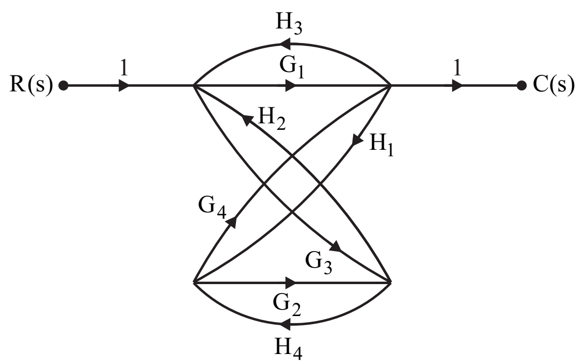
Hence, the system is over-damped.

The settling time for an over-damped system is given by

$$T_s=\frac{4}{\xi\omega_n-\omega_n\sqrt{\xi^2-1}}=\frac{4}{4-2\sqrt{2}}=3.41$$ **Ans.**

Note : Settling time of a system for an input of $u(t)$ and $5u(t)$ are same.

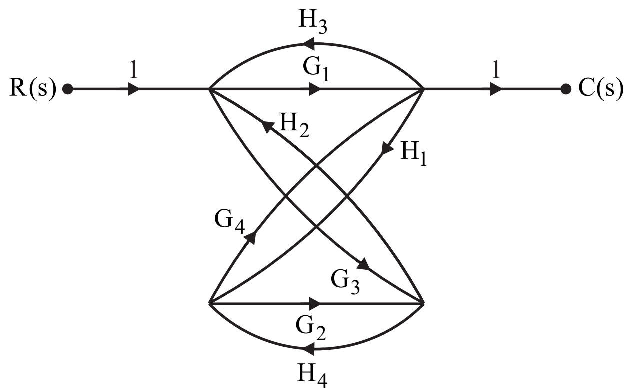
Q.5 Consider the signal flow graph shown in below figure.



The total number of individual loops are ________.

Ans. **6 to 6 (6)**

Sol.



The loops are

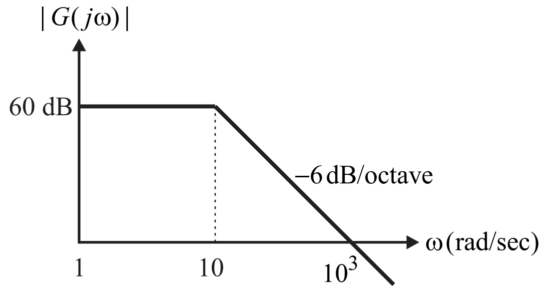
$$L_1 = G_1H_3,\ \ L_2 = G_2H_4$$

$$L_3 = G_4H_1,\ \ L_4 = G_3H_2$$

$$L_5 = G_1H_1G_2H_2,\ \ L_3 = G_3H_3G_4H_4$$

Hence, there are 6 loops in system.

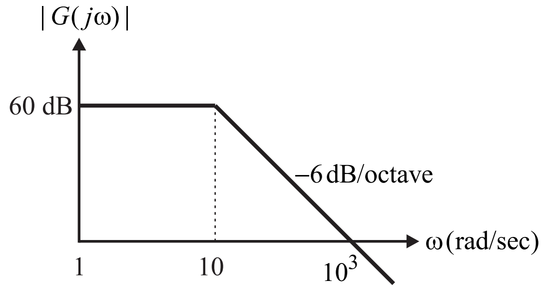
Q.6 The asymptotic magnitude plot for the open loop transfer function is shown in below figure. For unit step input the steady state error is



(A) $e_{ss} = \frac{1}{101}$ (B) $e_{ss} = \frac{1}{1001}$ (C) $e_{ss} = \frac{1}{110}$ (D) $e_{ss} = \frac{1}{1010}$

Ans. (B)

Sol.



Since, there exists only one slope of – 6 dB/octave that too after a corner frequency of 10 rad/sec

Also, -6 dB/octave $= -20$ dB/decade

$\Rightarrow$ Denominator is of form $\left(1 + \frac{s}{10}\right)$

Also, since at initial frequencies, the region is flat and non-zero, there exists some K

$$\Rightarrow \qquad G(s)H(s) = \frac{K}{\left(1+\frac{s}{10}\right)}$$

At $\omega = 1$ rad/sec,

Since $\omega < 10$ rad/sec $=$ corner frequency

$\Rightarrow$ Pole does not have any affect and only K is effective.

$$\Rightarrow \qquad 20\log K = 60$$

$$\Rightarrow \qquad K = 1000$$

$$\Rightarrow \qquad G(s)H(s) = \frac{1000}{\left(1+\frac{s}{10}\right)}$$

To calculate e_{ss} for unit step input, we calculate displacement error coefficient

$$k_P = \lim_{s\to 0} = \frac{1000}{\left(1+\frac{s}{10}\right)} = 1000$$

$$\Rightarrow \qquad e_{ss} = \frac{1}{1+k_p} = \frac{1}{1001}$$ **Ans.**

Hence, the correct option is (B).

Q.7 If $\omega_{gc} = \omega_{pc} = 10\,\text{rad/sec}$ for the given open loop transfer function of $G(s) = \frac{10(1-0.1s)}{s(1+0.1s)}$, then the phase margin and the gain margin are respectively.

(A) 0^0 and 6 dB (B) 45^0 and 0 dB (C) 45^0 and 6 dB (D) 0^0 and 0 dB

Ans. (D)

Sol.

$$\omega_{pc} = 10 \text{ rad/sec}$$

$$\text{GM} = -20\log|G(j\omega)|_{\omega=\omega_{pc}}$$

$$= -20\log \frac{10\times\sqrt{1^2+(0.1\times 10)^2}}{10\times\sqrt{1^2+(0.1\times 10)^2}}$$

$$= -20\log 1 = 0 \text{ dB}$$

$$\omega_{gc} = 10 \text{ rad/sec}$$

$$\text{PM} = 180^0 + \angle G(j\omega)|_{\text{at }\omega=\omega_{gc}}$$

$$= 180^0 - 90^0 - \tan^{-1}(0.1\omega_{gc}) - \tan^{-1}(0.1\omega_{gc})$$

$$= 180^0 - 90^0 - \tan^{-1}1 - \tan^{-1}1$$

$$= 0^0$$

Note : As $\omega_{pc} = \omega_{gc}$; the system is marginally stable and therefore GM = 0 dB and PM = 0^0.

Q.8 The first element of the each of the rows of a Routh-Hurwitz stability test showed the sign as follows :

Row :	I	II	III	IV	V	VI	VII	VIII
Signs :	+	–	–	+	+	+	–	–

The number of roots of the system lying in the left half of s-plane is

(A) 2 (B) 3 (C) 4 (D) 5

Ans. (C)

Sol. When there are N rows in the Routh Hurwitz table, The order of characteristic equation is $N - 1$

Hence, there are a total of $N - 1$ poles present in the closed system

Here, $N = 8$

Hence, $N - 1 = 7$ poles are present in total

Row	I	II	III	IV	V	VI	VII	VIII
Signs	+	–	–	+	+	+	–	–
	Sign change		Sign change			Sign change		

Since, the number of sign changes in 1st column of each row tells the number of poles in right half of the s plane

$\Rightarrow$ Total number of poles on right half = Total number of sign changes = 3

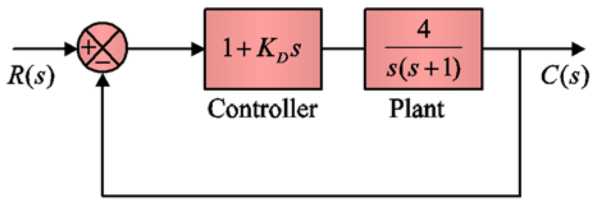
$\Rightarrow$ Total number of poles left side

= Total number of poles – Total number of poles on right side

$$= 7 - 3 = 4 \text{ Left hand poles}$$

Hence, the correct option is (C).

Q.9 A block diagram of a system with derivative controller is shown below.

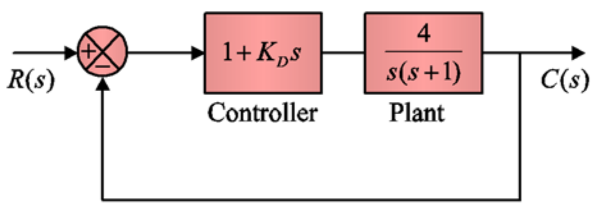


In order to make $\xi = 0.5,$ using controller for unit step response, the value of K_D will be

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2

Ans. (A)

Sol.

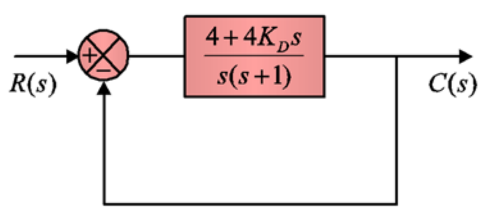


From diagram, we get

$$G'(s) = G_{\text{Controller}}(s) \times G_{\text{Plant}}(s)$$

$$= \frac{4 + 4K_D s}{s(s+1)}$$

The system becomes



Hence, closed loop transfer function

$$T(s) = \frac{G'(s)}{1 + G'(s)} = \frac{4 + 4K_D s}{s^2 + (4K_D + 1)s + 4}$$

The characteristic equation is

$$s^2 + (4K_D + 1)s + 4 = 0$$

Comparing above equation with standard characteristic equation

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

We get, $\omega_n^2 = 4$

$\Rightarrow$ $\omega_n = 2$

Also, $2\xi\omega_n = (4K_D + 1)$

$\Rightarrow$ $4\xi = 4K_D + 1$

Since, we need to make $\xi = 0.5$

$\Rightarrow$ $4 \times 0.5 = 4K_D + 1$

$\Rightarrow$ $4K_D = 1$

$\Rightarrow$ $K_D = \dfrac{1}{4}$ **Ans.**

Hence, the correct option is (A).

Q.10 The state equations of a system are given below:

$$\dot{X}_1 = X_1 + X_2 + U$$

$$\dot{X}_2 = -X_2 \text{ and } Y = X_1$$

The system is said to be

(A) Controllable as well as Observable (B) Only Controllable

(C) Only Observable (D) Neither Controllable nor Observable

Ans. (C)

Sol. Converting the given equations in standard matrix form

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} [U]$$

$$Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

Since, standard equations are

$$[\dot{X}] = [A][X] + [B][U]$$

$$[Y] = [C][X]$$

We get, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

and $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

Check for controllability

$$AB = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$Q_C = [B : AB] = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$|Q_C| = 1$ Uncontrollable

Check for observability

$$C^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix} A^T = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

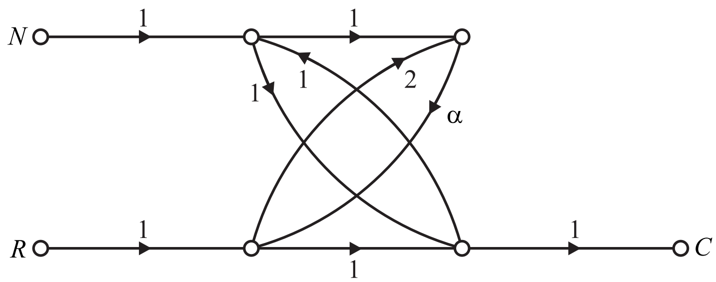
$$Q_0 = \left[C^T : C^T A^T \right] = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$|Q_0| = 1$$

Hence, Q_0 is non-singular also the rank of Q_0 is equal to rank of A hence Observable.

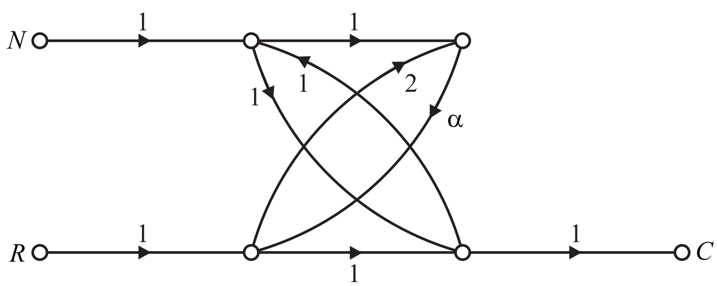
Q.11 to Q.30 carry two marks each

Q.11 The signal flow graph given below has an output C and is excited by a step input at node R and is also corrupted by a noise N. The value of α so that interference at output due to noise is minimum will be ____________.



Ans. **1 to 1 (1)**

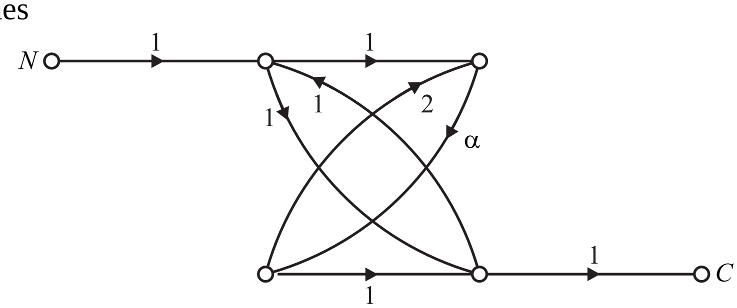
Sol.



To find effect of Noise N on output C, we find transfer function $\frac{C}{N}$ using superposition and taking

$R = 0$

Hence, SFG becomes



Using mason's gain formula, we calculate forward path gains

$$P_1 = 1\times 1\times \alpha \times 1\times 1 = \alpha$$

$$P_2 = 1\times 1\times 1 = 1$$

The loops are

$$L_1 = 1\times 1 = 1\ ,\quad L_2 = 2\times \alpha = 2\alpha$$

$$L_3 = 1\times \alpha \times 1\times 1 = \alpha$$

Since, Δ_k is associated path factor with path P_k

$$\Delta_1 = 1-(0) = 1$$

$$\Delta_2 = 1-(2\alpha) = 1-2\alpha$$

Total path factor

$\Delta = 1-$(sum of individual loops) + (sum of product of two non touching loops)

Since L_1 and L_2 are non-touching loops

$$\Delta = 1-(1+\alpha+2\alpha)+1\times 2\alpha = -\alpha$$

Hence

$$\frac{C}{N} = \frac{\Sigma P_k \Delta_k}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\frac{C}{N} = \frac{\alpha.1+1(1-2\alpha)}{-\alpha} = \frac{1-\alpha}{-\alpha}$$

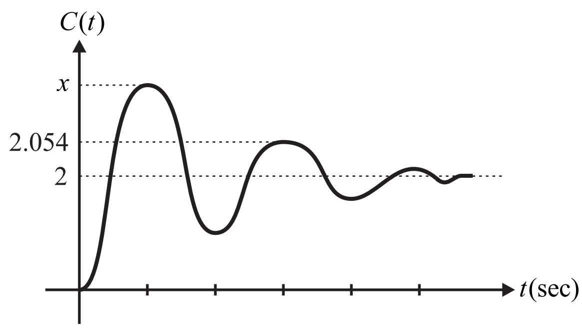
Hence $\frac{C}{N}$ will be minimum (zero)

When $\frac{C}{N} = \frac{\alpha-1}{\alpha} = 0$

Hence, $1-\alpha = 0$

$\Rightarrow$ $\boxed{\alpha = 1}$ **Ans.**

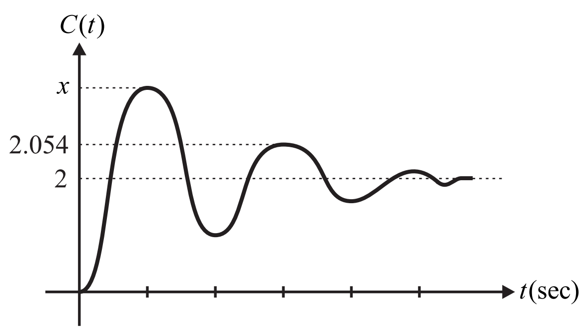
Q.12 Given below is a response of an under damped system excited by an input of $2u(t)$.



The value 'x' shown in above figure is __________.

Ans. **2.4 to 2.8 (2.6)**

Sol.



From figure, we can see that the steady state value output is 2.
Also, we see the value of 2nd maximum peak overshoot (M.P.O) is 2.054

Normalized 2nd M.P.O = $= \frac{2.054-2}{2}$

$$e^{\frac{-3\xi\pi}{\sqrt{1-\xi^2}}} = 0.027 \quad ...(i)$$

Taking cube root

$$e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} = 0.3$$

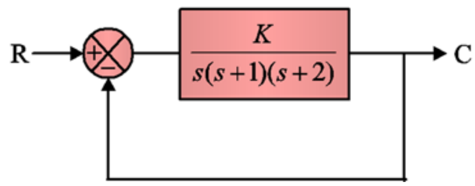
As we know,

$$e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} = \text{Normalised 1st M.P.O}$$

$\Rightarrow$ $0.3 = \frac{x-2}{2}$

$\Rightarrow$ $\boxed{x = 2.6}$ **Ans.**

Q.13 Consider negative feedback system shown in below figure.



The maximum value of K for which system is overdamped will be ________.

Ans. 0.36 to 0.40 (0.38)

Sol. $G(s) = \frac{K}{s(s+1)(s+2)}$

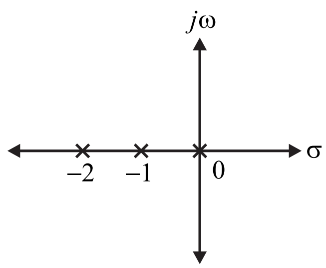
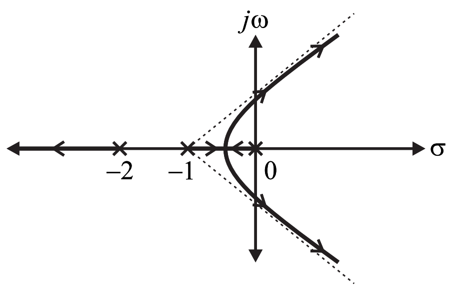


Fig. Third order system

Concept of Root Locus :

1. Plotting root locus
2. Number of branches $= P = 3$
3. Number of asymptotes $= P - Z = 3$
4. Angle of asymptotes $= 60^0, 180^0, 300^0$
5. Centroid $= \sigma = \frac{0-1-2}{3} = -1$

The approximate root locus is as follows



From figure, we can see that poles exist at real axis for some values of K and leave the real axis for some values of K

As, root locus starts at real axis, as K increases it leaves the real axis.

Also, as we know, for an overdamped system its poles lie on real axis.

So the maximum value of K for which the system is overdamped is value of K at break away point.

For Break point,

We use equation $1+G(s)=0$ and calculate values of s for which $\frac{dK}{ds}=0$

$$1+\frac{K}{s(s+1)(s+2)}=0$$

$$K=-s(s^2+3s+2)$$

$$K=-(s^3+3s^2+2s) \quad \text{...(i)}$$

$$\frac{dK}{ds}=-(3s^2+6s+2)$$

For break point $\frac{dK}{ds}=0$

Hence, $s=-0.422,-1.577$

But $s=-1.577$ does not exist on root locus as from root locus we find that breakaway point exists between 0 and -1.

Hence, the break point is $s=-0.422$.

Hence, from equation (i),using $s=-0.422$

$$K=0.38$$

Ans.

Q.14 A first order matrix differential equation of a system is given as $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 2\end{bmatrix}u$,

$y=[1 \quad 1]\begin{bmatrix}x_1\\ x_2\end{bmatrix}+[1]u$

The transfer function of the system is

(A) $\frac{2}{s+2}$ (B) $\frac{s+4}{s+2}$ (C) $\frac{2}{s+1}$ (D) $\frac{s+3}{s+1}$

Ans. (B)

Sol. Given : $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 2\end{bmatrix}u$, $y=[1 \quad 1]\begin{bmatrix}x_1\\ x_2\end{bmatrix}+[1]u$ ...(i)

State equation is given by,

$$\dot{x}=Ax+Bu \quad \text{...(ii)}$$

Output state equation is given by,

$$y=Cx+Du \quad \text{...(iii)}$$

On comparing equation (i), (ii) and (iii), we get

$$A=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}, \quad B=\begin{bmatrix}0\\ 2\end{bmatrix}, \quad C=[1 \quad 1], \quad D=[1]$$

(i) Transfer function is given by,

$$T(s) = C\left[sI - A\right]^{-1} B + D$$

$$\left[sI - A\right] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$$

$$\left[sI - A\right]^{-1} = \frac{\text{Adj}\left[sI - A\right]}{\left|sI - A\right|}$$

$$\text{Adj}\left[sI - A\right] = \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$$

$$\left|sI - A\right| = s(s+3) + 2 = s^2 + 3s + 2$$

$$[sI - A]^{-1} = \begin{bmatrix} \frac{s+3}{s^2+3s+2} & \frac{1}{s^2+3s+2} \\ \frac{-2}{s^2+3s+2} & \frac{s}{s^2+3s+2} \end{bmatrix}$$

$$T(s) = \left[1 \quad 1\right] \begin{bmatrix} \frac{s+3}{s^2+3s+2} & \frac{1}{s^2+3s+2} \\ \frac{-2}{s^2+3s+2} & \frac{s}{s^2+3s+2} \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} + [1]$$

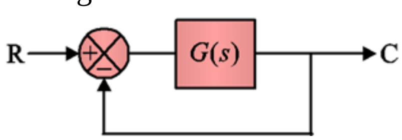
$$T(s) = \left[1 \quad 1\right] \begin{bmatrix} \frac{2}{s^2+3s+2} \\ \frac{2s}{s^2+3s+2} \end{bmatrix} + [1]$$

$$T(s) = \frac{2+2s}{s^2+3s+2} + 1 = \frac{2(s+1)}{(s+1)(s+2)} + 1 = \frac{2}{s+2} + 1 = \frac{s+4}{s+2}$$

$$T(s) = \frac{s+4}{s+2}$$ **Ans.**

Hence, the correct option is (B).

Q.15 Consider the system shown in below figure.



Given : Gain crossover frequency = 10 rad/sec

Phase crossover frequency = 20 rad/sec

Whereas $G(s)$ has a transfer function with no zero and one Right-hand pole.

Then, which of the statements is correct about the closed system?

(A) Routh-Hurwitz table has exactly one R.O.Z.

(B) Routh-Hurwitz table has exactly 1 sign change in the 1st column.

(C) The Nyquist plot of system will have 2 encirclement around –1+ $j0$.

(D) The Nyquist plot of system will have 1 encirclement around –1+ $j0$.

Ans. **(D)**

Sol. **Given :** $\omega_{gc} = 10$ rad/sec, $\omega_{pc} = 20$ rad/sec

Since, $\omega_{pc} > \omega_{gc}$

Hence, the closed loop system is absolutely stable

Option A

Since, presence of one ROZ tells that system is marginally stable but, our system is absolutely stable.

Hence, option A is wrong.

Option B

One sign change in 1st column implies presence of 1 right hand closed loop pole. But, since our system is stable, hence no closed loop poles lie on right hand side.

Hence, option B is wrong.

Concept of Nyquist plot,

$$N = P - Z$$

Where N is encirclement (anti clock wise)

$P =$ Open loop poles on right hand side

$Z =$ Closed loop poles on right hand side

Option C

$$N = \pm 2,\; P = 1 \Rightarrow Z = P - N$$

For $N = +2$ $Z = -1$ (not possible)

For $N = -2$ $Z = 3$ (means unstable system)

Hence option C is wrong

Option D

$$N = \pm 1,\; P = 1 \Rightarrow Z = P - N$$

For $N = -1 \Rightarrow Z = 2$ (means unstable)

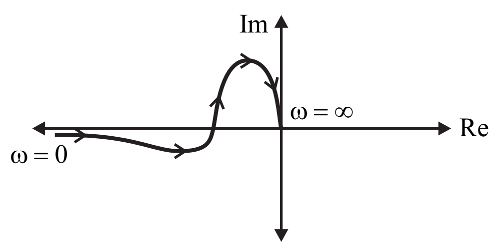
For $N = 1 \Rightarrow Z = 0$ (stable system)

Since is not specified whether encirclement was clock wise or anti clock wise.

But $N = 1$ gives stable system which satisfies the description of system

Hence, correct option is (D).

Q.16 Figure shown below is a polar plot.



Some open loop transfer functions are given below.

1. $G(s)H(s) = \dfrac{s+1}{s^2(s+4)(s+6)}$

2. $G(s)H(s) = \dfrac{(-s-1)}{(-s-4)(-s-6)}$

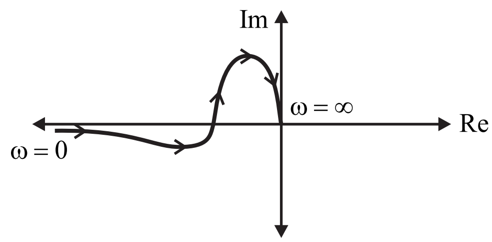
3. $G(s)H(s) = \dfrac{s+10}{s^2(s+1)(s+2)}$

Its probable transfer function is/are

(A) 3 only (B) 2 and 3 (C) 1 and 2 (D) 1, 2 and 3

Ans. (C)

Sol.



1. $G(s)H(s)=\dfrac{s+1}{s^2(s+4)(s+6)}$

$$\angle G(s)H(s)=-180^0+\tan^{-1}\omega-\tan^{-1}\frac{\omega}{4}-\tan^{-1}\frac{\omega}{6}$$

Hence at $\omega=0$

$$\angle G(0)H(0)=-180^0$$

At $\omega=\infty$ $\quad \angle G(\infty)H(\infty)=-270^0$

Since there exists a point in polar plot where x-axis is cut i.e. imaginary part of GH is zero

$$G(j\omega)H(j\omega)=\frac{j\omega+1}{-\omega^2(j\omega+4)(j\omega+6)}$$

$\Rightarrow \quad I_m[G(j\omega)H(j\omega)]=0$

Rationalizing

$$G(j\omega)H(j\omega)=\frac{(j\omega+1)(j\omega-4)(j\omega-6)}{-\omega^2(\omega^2+16)(\omega^2+36)}$$

Hence $\quad I_m[(j\omega+1)(j\omega-4)(j\omega-6)]=0$

$$I_m[(j\omega+1)(-\omega^2+24-10j\omega)]=0$$

$$-\omega^3+24\omega-10\omega=0$$

$\Rightarrow \quad \omega=\sqrt{14}$ rad/sec

Since, there exists an ω for which plot cuts imaginary axis. Hence, this transfer satisfies the polar plot condition and is a probable transfer function.

2. $G(s)H(s)=\dfrac{-s-1}{(-s-4)(-s-6)}$

$$\angle G(s)H(s)=180+\tan^{-1}\omega-\left(180+\tan^{-1}\frac{\omega}{4}\right)-\left(180+\tan^{-1}\frac{\omega}{6}\right)$$

$$=-180^0+\tan^{-1}\omega-\tan^{-1}\frac{\omega}{4}-\tan^{-1}\frac{\omega}{6}$$

Since expression of $\angle GH$ is same as in previous transfer function so, this transfer function also exhibits similar polar plot and hence is a probable transfer function.

3. $G(s)H(s)=\dfrac{s+10}{s^2(s+1)(s+2)}$

$$\angle G(s)H(s)=-180^0+\tan^{-1}\frac{\omega}{10}-\tan^{-1}\omega-\tan^{-1}\frac{\omega}{2}$$

At $s=0$ $\quad \angle G(0)H(0)=-180^0$

At $s=\infty$ $\quad \angle G(\infty)H(\infty)=-270^0$

$$G(j\omega)H(j\omega)=\frac{j\omega+10}{-\omega^2(j\omega+1)(j\omega+2)}$$

Rationalizing denominator

$$\frac{(j\omega+10)(j\omega-1)(j\omega-2)}{-\omega^2(\omega^2+1)(\omega^2+4)}$$

For point where polar plot cuts imaginary axis

$$I_m[(j\omega+10)(-\omega^2-3j\omega+2)]=0$$

$$-\omega^3+2\omega-30\omega=0$$

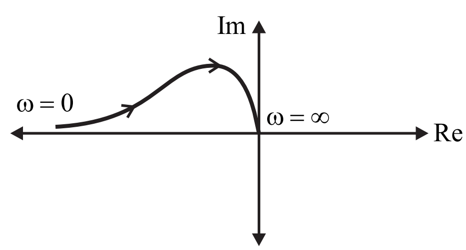
$\Rightarrow$ $\quad -\omega(\omega^2+28)$

$\Rightarrow$ $\quad \omega=0,\ -j\sqrt{28},\ j\sqrt{28}$

Since values of ω are imaginary

Hence, the polar plot does not cut real axis.

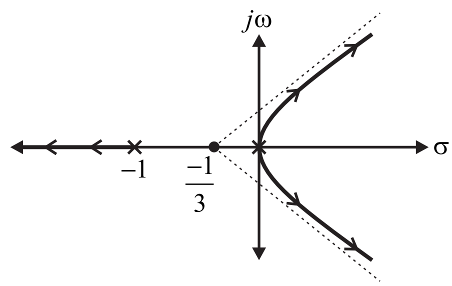
The plot for this transfer function will be



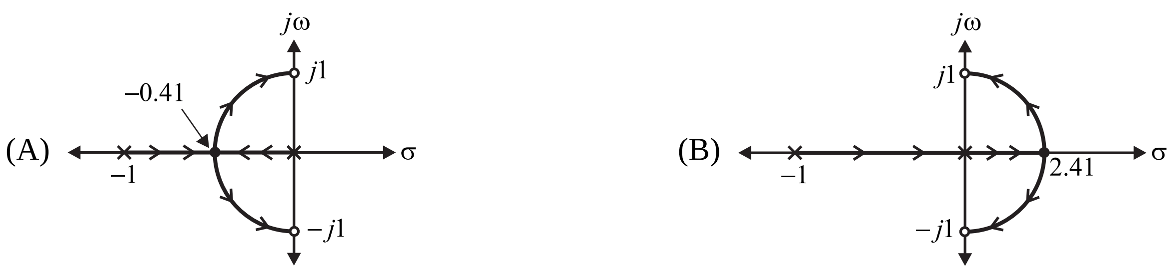
Hence transfer function (i) and (ii) are probable ones for polar plot

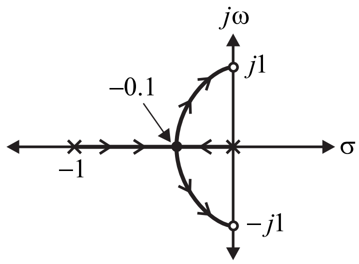
Hence, the correct option is (C).

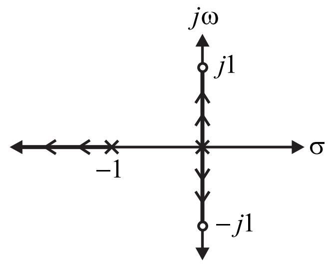
Q.17 A root locus diagram for a system $G(s)H(s)$ is shown in below figure.



When three zeros $s=0,\pm j1$ are added to the system the new root locus will become.

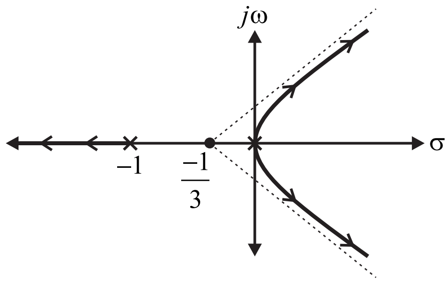


(C) 

(D) 

Ans. (A)

Sol.



Observations :

1. There are no zeroes.
2. There are 3 asymptotes.
   Hence, there are 3 poles in total.
3. Root locus leaves x axis at $s = 0$.
   Break-away point exists at $s = 0$.
   There exists a double pole at origin.

Transfer function is

$$G(s)H(s) = \frac{K}{s^2(s+1)}$$

Now adding zeroes $s = 0$, $\pm j1$

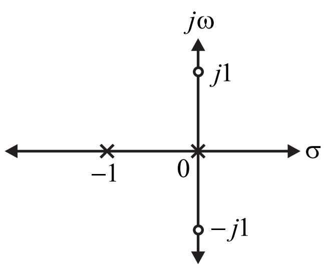
New open loop transfer function becomes

$$G(s)H(s) = \frac{K(s^2+1)}{s(s+1)}$$

For new root locus

Number of Branches = $Z = P = 2$

Number of Asymptotes $= P - Z = 0$



Since, for existence of root locus on real axis, total no. of poles and zeros to right of that point must be odd.

Hence, root locus exists between 0 and –1.

Also, since root locus ends at a zero, it must leave x axis at some point

Breakaway point exists

Calculation of breakaway point :

$$K = \frac{-s(s+1)}{s^2+1} = \frac{-s^2-s}{s^2+1} = \frac{-(s^2+s)}{(s^2+1)}$$

$$\frac{dK}{ds} = 0$$

$$\frac{dK}{ds} = \frac{(2s+1)(s^2+1)-(2s)(s^2+s)}{(s^2+1)^2}$$

$$2s^3+2s+s^2+1-2s^3-2s^2 = 0$$

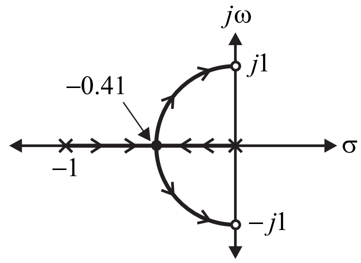
$$-s^2+2s+1=0$$

$$s = 2.41,\ -0.41$$

But, breakaway point exists between two poles. Hence, it must exist between 0 and – 1

$\Rightarrow$ $s = -0.41$

Hence, its root locus is



Hence, the correct option is A.

Q.18 Consider the two open loop transfer functions shown below

$$G_1(s) = \frac{1}{(s+1)(s+2)},\quad G_2(s) = \frac{100}{(s+9)(s+10)}$$

The phase margins for $G_1(s)$ and $G_2(s)$ are respectively.

(A) ∞, ∞ (B) ∞, 143.2^0 (C) ∞, 43.2^0 (D) -43.2^0, ∞

Ans. (B)

Sol. $$G_1(s) = \frac{1}{(s+1)(s+2)},\quad G_2(s) = \frac{100}{(s+9)(s+10)}$$

For phase margin calculation, we need gain crossover frequency

For $G_1(s)$,

$$G_1(s) = \frac{1}{(j\omega+1)(j\omega+2)}$$

$$|G_1(j\omega)| = \frac{1}{(\sqrt{\omega^2+1})(\sqrt{\omega^2+4})}$$

Gain crossover frequency ω_{GC}

$$|G_1(j\omega_{GC})| = \frac{1}{\sqrt{\omega_{GC}^2+1}\sqrt{\omega_{GC}^2+4}} = 1$$

$\Rightarrow \quad (\omega_{GC}^2+1)(\omega_{GC}^2+4)=1$

$\Rightarrow \quad (\omega_{GC}^2)^2+5(\omega_{GC}^2)+4=1$

$\Rightarrow \quad (\omega_{GC}^2)^2=-0.69\,,\ -4.3$

Since ω_{GC} is a positive value hence, for G_1, ω_{GC} does not exist.

$\Rightarrow \quad \boxed{PM=\infty}$

For $G_2(s)$

$$G_2(j\omega)=\frac{100}{(j\omega+9)(j\omega+10)}$$

$$|G_2(j\omega)|=\frac{100}{\sqrt{\omega^2+81}\sqrt{\omega^2+100}}$$

Gain crossover frequency ω_{GC}

$$|G_2(j\omega_{GC})|=\frac{100}{\sqrt{\omega_{GC}^2+81}\sqrt{\omega_{GC}^2+100}}=1$$

$\Rightarrow \quad (10)^4=(\omega_{GC}^2+81)(\omega_{GC}^2+100)$

$\Rightarrow \quad (10)^4=(\omega_{GC}^2)^2+181(\omega_{GC}^2)+8100$

$\Rightarrow \quad (\omega_{GC}^2)^2+181(\omega_{GC}^2)-1900=0$

$\Rightarrow \quad \omega_{GC}^2=9.95,-190.95$

But, since ω_{GC} is a real and positive quantity

$\Rightarrow \quad \omega_{GC}=\sqrt{9.95}=3.15$ rad/sec

$$\angle G_2(j\omega)|=-\tan^{-1}\frac{\omega}{9}-\tan^{-1}\frac{\omega}{10}$$

$$\angle G_2(j\omega_{GC})|=-\tan^{-1}\frac{3.15}{9}-\tan^{-1}\frac{3.15}{10}$$

$$=-36.77^0$$

Hence, phase margin $PM=180^0+\angle GH(j\omega_{GC})$

$$=180^0-36.77^0$$

$$\boxed{PM=143.23^0}$$

Hence, the correct option is (B).

Concept :

ω	$\lvert G_1(j\omega)\rvert$	$\lvert G_2(j\omega)\rvert$
0	0.5	1.11
∞	0	0

Here, we see $|G_1(j\omega)|$ is never equal to 1.

Hence, ω_{GC} does not exist.

We see $|G_2(j\omega)|$ is equal to 1 for some ω.

Hence, ω_{GC} exists.

Q.19 The closed loop transfer function of a system is given below $T(s)=\frac{s+1}{s^2+3s+1}$. The steady state error for its unity negative feedback system for input $tu(\text{t})$ will be __________.

Ans. 2 to 2 (2)

Sol.
$$T(s)=\frac{s+1}{s^2+3s+1}$$

For conversion to unity negative feedback system

$$T(s)=\frac{G(s)}{1+G(s)}=\frac{s+1}{s^2+3s+1}$$

$$G(s)=\frac{N}{D-N}$$

$$G(s)=\frac{s+1}{s^2+3s+1-(s+1)}$$

$$\Rightarrow \quad \boxed{G(s)=\frac{s+1}{s(s+2)}}$$

Steady state error for an input $tu(t)$ is given by

$$e_{ss}=\frac{1}{k_v}$$

Where, $$k_v=\lim_{s\to 0} sG(s)=\lim_{s\to 0}\frac{s(s+1)}{s(s+2)}=\frac{1}{2}$$

$$\Rightarrow \quad e_{ss}=\frac{1}{k_v}=2$$ **Ans.**

Q.20 A proportional derivative controller $G_d(s)=10(1+0.1s)$ is added in the forward path of the second order system $\frac{C(s)}{R(s)}=\frac{20}{s^2+10s+20}$ The value of new damping coefficient is

(A) 1 (B) 2.5 (C) 0.5 (D) 0.75

Ans. (A)

Sol.
$$\frac{C(s)}{R(s)}=\frac{20}{s^2+10s+20}$$

Let $G(s)$ be open loop transfer function taking unity negative feedback.

Then,

$$\frac{G(s)}{1+G(s)}=\frac{20}{s^2+10s+20}$$

Taking reciprocal

$$\frac{1+G(s)}{G(s)}=\frac{s^2+10s+20}{20}$$

Subtracting 1 from both sides

$$\frac{1}{G(s)}=\frac{s^2+10s}{20}$$

$\Rightarrow \qquad G(s) = \dfrac{20}{s^2 + 10s}$

When a PD controller $G_d(s) = 10(1 + 0.1s)$ is added to the forward path

$$G(s) = \frac{20}{s^2 + 10s}$$

New open loop transfer function becomes

$$G'(s) = G(s) \times G_d$$

$$= (10 + s) \times \frac{20}{s(s+10)} = \frac{20s + 200}{s^2 + 10s}$$

Hence, closed loop transfer function becomes

$$T(s) = \frac{G'(s)}{1 + G'(s)}$$

$$\frac{\dfrac{20s+200}{s^2+10s}}{1 + \dfrac{20s+200}{s^2+10s}} = \frac{20s+200}{s^2+30s+200}$$

Hence, the characteristic equation is $s^2 + 30s + 200$

Comparing with standard characteristic equation

$$s^2 + 2\xi\omega_n s + \omega_n^2$$

We get $\qquad \omega_n^2 = 200$

$\Rightarrow \qquad \omega_n = 14.14$

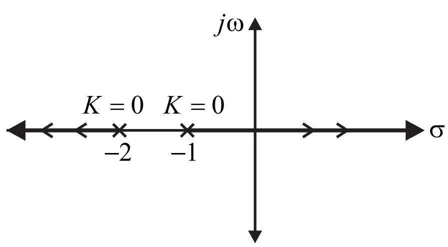
Also $\qquad 2\xi\omega_n = 30$

$\Rightarrow \qquad 2 \times 14.14 \times \xi = 30$

Hence, $\qquad \boxed{\xi = 1.06}$ **Ans.**

Hence, the correct option is (A)

Q.21 The root locus of a negative unity feedback system is shown in the figure.



The closed-loop transfer function the system is

(A) $\dfrac{C(s)}{R(s)} = \dfrac{K}{(s+1)(s+2)}$

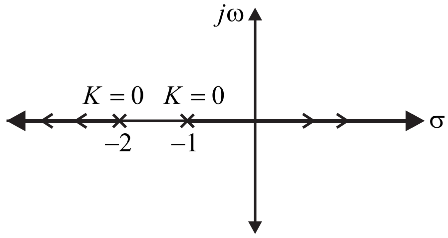
(B) $\dfrac{C(s)}{R(s)} = \dfrac{-K}{(s+1)(s+2)+K}$

(C) $\dfrac{C(s)}{R(s)} = \dfrac{-K}{(s+1)(s+2)-K}$

(D) $\dfrac{C(s)}{R(s)} = \dfrac{K}{(s+1)(s+2)+K}$

Ans. **(C)**

Sol.



Total number of poles and zeros on right of $s=-1$ are 0 (even)

But, Root locus exists there (from figure)

Similarly, for a point on left of $s=-2$, for example for $s=-3$, the total no. of poles and zeroes on the right of it is even.

Still, root locus exist there. (from figure)

Hence, we may conclude that the figure is actually on inverse root locus.

We see that there are two poles at $s=-2$ and $s=-1$

For $0<K<\infty$

Inverse root locus gives transfer function of form

$$G(s)=\frac{-K}{(s+1)(s+2)}$$

$$\frac{G(s)}{1+G(s)}=\frac{-K}{(s+1)(s+2)-K}$$

Hence, the correct option is (C).

Q.22 Open loop transfer function of a system is given by $G(s)H(s)=\frac{(s+1)(s+2)^2(s+3)^3}{s^2}$.

The number of infinite magnitude half circles in its Nyquist plot will be_______.

Ans. **6 to 6 (6)**

Sol. $$G(s)H(s)=\frac{(s+1)(s+2)^2(s+3)^3}{s^2}$$

For a Nyquist plot, infinite magnitude half circles are formed only when :

(i) Poles exist at origin

If the number of poles which exist at origin is P, then P half circles are contributed by it which encircle in clock wise direction.

Here $P=2$

Two infinite magnitude half circles are contributed in clockwise direction.

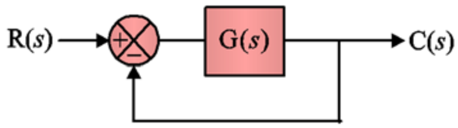
(ii) When the total number of zeroes is greater than the total number of poles.

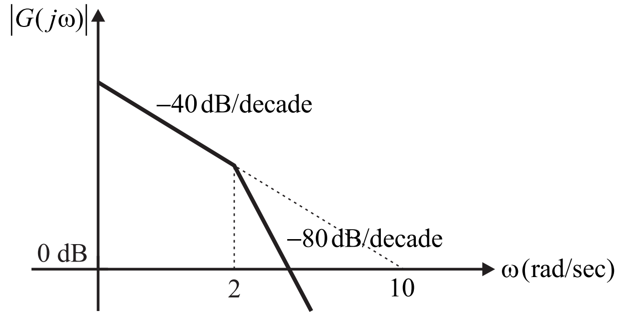
If there are Z number of zeroes and P number of poles, then for $Z>P$, $Z-P$ infinite magnitude half circles are contributed by it in clockwise direction here $Z=6$, $P=2$

Hence, 4 infinite magnitude half circles are contributed by it.

Hence, total number of infinite magnitude half circles is 4 + 2 = 6 **Ans.**

Q.23 Consider the bode plot of function $G(s)$ shown in figure where system is represented as below. The steady state error for an input of $t^2u(t)$ will be ________.

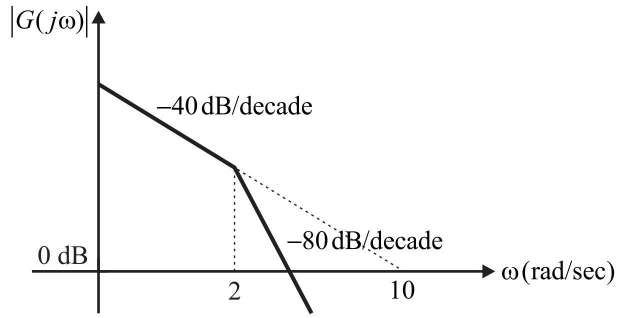




(A) 0.01 (B) 0.02 (C) 0 (D) 0.2

Ans. (B)

Sol.



From figure, we can see that the plot has a initial slope of -40 dB/decade which implies presence of 2 poles of origin.

Also, we see that of after $\omega = 2\,\text{rad/sec}$ we get a slope of $-80\,\text{dB/decade}$

Which implies presence of two additional pole.

Hence open loop transfer function is of form

$$G(s) = \frac{K}{s^2\left(1+\frac{s}{2}\right)^2}$$

From bode plot

$$20\log K - 40\log 10 = 0$$

$$K = 100$$

Hence,

$$G(s) = \frac{100}{s^2\left(1+\frac{s}{2}\right)^2}$$

For an input of $\frac{t^2}{2}u(t)$ the steady state error is given by $e_{ss} = \frac{1}{k_a}$

Where acceleration error coefficient k_a is given by

$$k_a = \lim_{s\to 0} s^2 G(s)$$

$$k_a = \lim_{s\to 0} \frac{100s^2}{s^2\left(1+\frac{s}{2}\right)^2}$$

$$k_a = 100$$

Hence, if the input is $\frac{t^2}{2}u(t)$ the steady state error is given by $\frac{1}{k_a}$

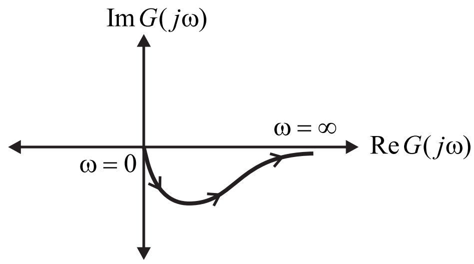
But, since input is $t^2u(t)$ the steady state error is $\frac{2}{k_a}$

$$e_{ss} = \frac{2}{100} = 0.02$$ **Ans.**

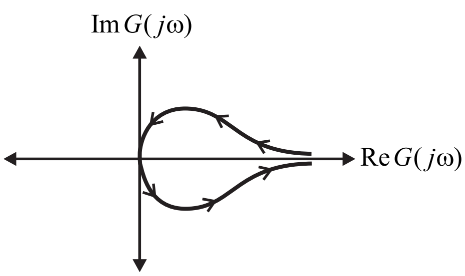
Q.24 The open loop transfer function $G(s)$ of a unity negative feedback system is of form $G(s) = \frac{N(s)}{2-s}$.

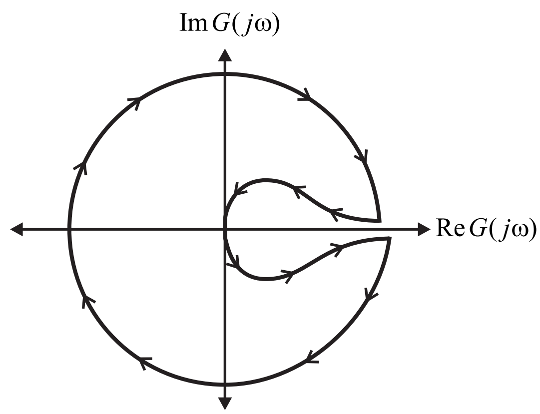
Where $N(s)$ is a polynomial of s with positive coefficient(s) and positive power(s).

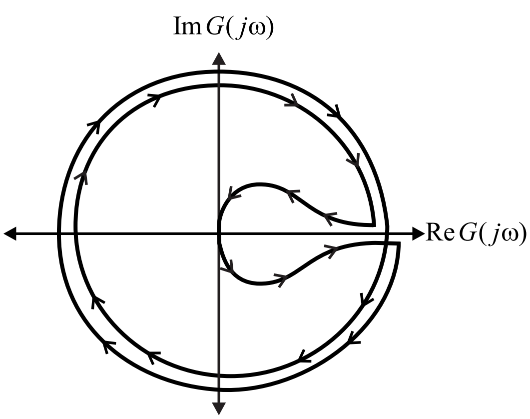
The polar plot representation of system is shown below

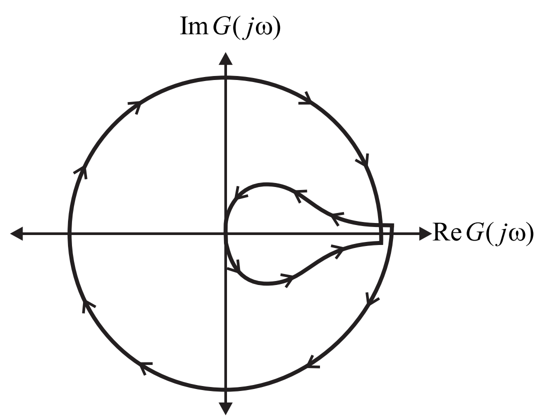


The Nyquist plot for the system will be

(A)


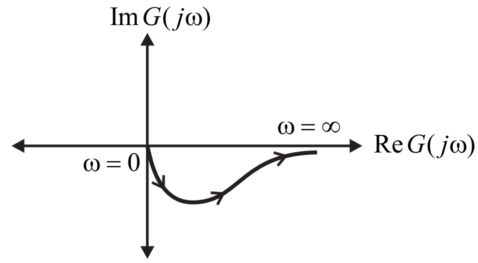
(B)


(C)


(D)


Ans. (B)

Sol.



Since $G(s) = \dfrac{N(s)}{2-s}$

$$\angle G(s) = \angle N(s) - \tan^{-1}\left(-\frac{\omega}{2}\right)$$

$$\angle G(s) = \angle N(s) + \tan^{-1}\left(\frac{\omega}{2}\right)$$

Since, we observe that at $\omega = 0$, $\angle G(s) = 270^0$

$$270^0 = \angle N(s) + \tan^{-1}(0)$$

$$\angle N(s) = 270^0$$

Since, there exists only one system which is capable of providing 270^0 angle even at $\omega = 0$ and is of polynomial from which is $N(s) = ks^3$

$\Rightarrow$ $$G(s) = \frac{N(s)}{2-s} = \frac{ks^3}{2-s}$$

$\Rightarrow$ $$\angle G(j\omega) = 270^0 + \tan^{-1}\frac{\omega}{2}$$

$$|G(j\omega)| = \frac{k\left|\omega^3\right|}{\sqrt{4+\omega^2}}$$

Also $|G(j\omega)|_{\omega=0} = 0$

Also $|G(j\omega)|_{\omega=\infty} = \infty$

Since, polar plot given in the question satisfies the phase and magnitude both.

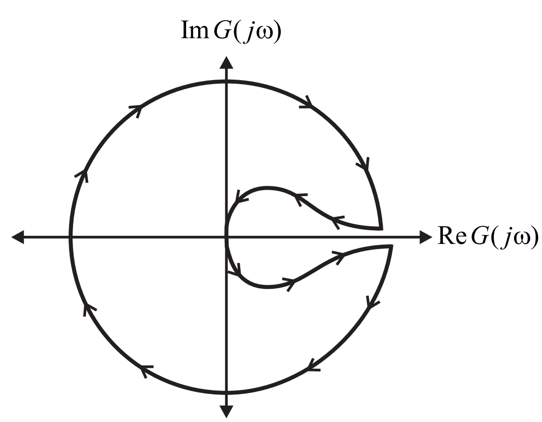
Hence, the transfer function

$$G(s) = \frac{ks^3}{2-s} \text{ is correct}$$

For the Nyquist plot, the total number of ∞ magnitude half circles (if Z is the number of zeroes in open loop transfer function and P is the number of poles in open loop transfer function) is $Z - P = 3 - 1 = 2$

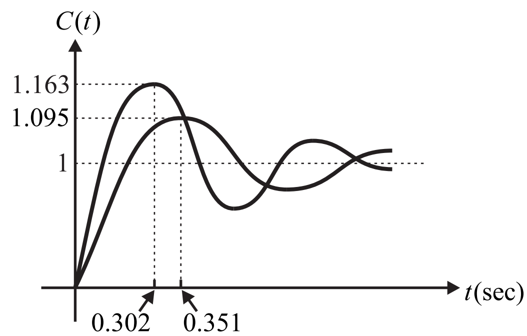
Hence, we have 2 clockwise ∞ magnitude half circles.

Hence, the Nyquist plot becomes



Hence, the correct option is (B).

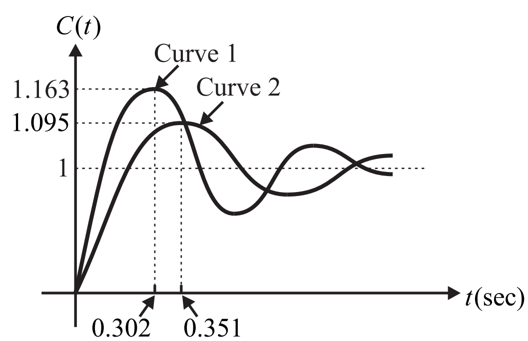
Q.25 Consider the plot of an underdamped system as shown in below figure. One of the plot represents the system with a proportional controller with gain K_p and other without it.



The undamped natural frequency for system with controller will be _______.

Ans. 11 to 13 (12.05)

Sol.



As we know proportional controller increases the damped frequency ω_d.

Hence, the peak time $\frac{\pi}{\omega_d}$ decreases.

Also, as ξ decreases when proportional controller is used. Hence, maximum peak overshoot increases.

So, curve 1 represents proportional controller output.

From curve 1 :

$$\text{M.P.O.} = \frac{1.163-1}{1} = 0.163 = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$$

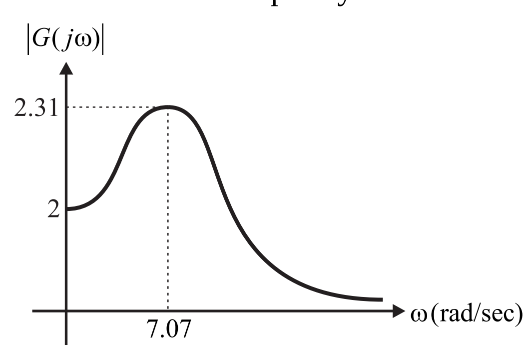
Hence, $\xi = 0.5$

Also, we find from curve that peak time of the output with controller is 0.302

Hence, $$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{\omega_n\sqrt{1-(0.5)^2}} = 0.302$$

Hence, $\omega_n = 12.05$ **Ans.**

Q.26 Frequency response of a second order underdamped system is shown in below figure

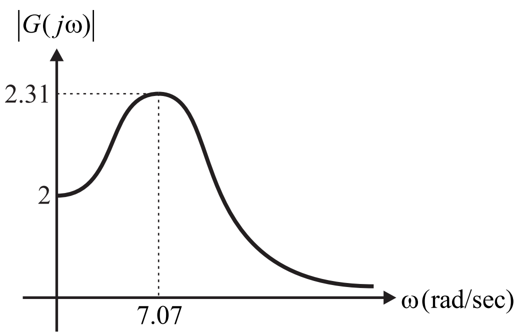


Closed loop transfer function of the system will be

(A) $\frac{100}{s^2+12s+50}$ (B) $\frac{200}{s^2+10s+200}$ (C) $\frac{200}{s^2+10s+100}$ (D) $\frac{100}{s^2+5s+50}$

Ans. (C)

Sol.



We see that from figure, $K = 2$

Also, peak is given by

$$\frac{K}{2\xi\sqrt{1-\xi^2}}$$

Hence, $2.31 = \frac{2}{2\xi\sqrt{1-\xi^2}}$

Hence, $\xi = 0.5, 0.866$

But, $\xi < 0.7$

Hence we take $\xi = 0.5$

Since, resonant frequency is given by

$$\omega_r = \sqrt{1-2\xi^2}\,\omega_n$$

From figure $\omega_r = 7.07$

Using $\xi = 0.5$

$$\omega_n = \frac{\omega_r}{\sqrt{1-2(0.5)^2}} = 10$$

Since the standard transfer function is given by

$$T(s) = \frac{K\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

Hence, $T(s) = \frac{200}{s^2+10s+100}$

Hence, the correct option is (C).

Q.27 The transfer function of a compensator is given below. The conditions under which it works as a Lag-lead compensator is

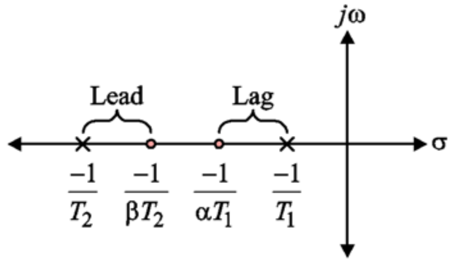
$$T(s) = \left(\frac{1+\alpha sT_1}{1+sT_1}\right)\left(\frac{1+\beta sT_2}{1+sT_2}\right)$$

(A) $\alpha > 1, \beta < 1, T_1 > T_2$ (B) $\alpha < 1, \beta > 1, T_1 < T_2$

(C) $\alpha < 1, \beta > 1, T_1 > T_2$ (D) $\alpha > 1, \beta > 1, T_1 > T_2$

Ans. **(C)**

Sol. For lag-lead compensator, pole zero plot would be as shown below



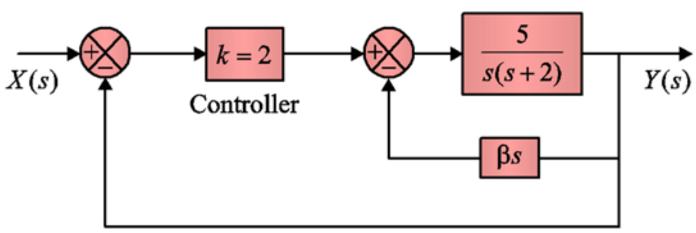
So we see, $\left|\frac{1}{\beta T_2}\right| < \left|\frac{1}{T_2}\right| \Rightarrow \beta > 1$

Similarly, $\left|\frac{1}{T_1}\right| < \left|\frac{1}{T_2}\right| \Rightarrow T_2 < T_1$

Similarly, $\left|\frac{1}{T_1}\right| < \left|\frac{1}{\alpha T_1}\right| \Rightarrow \alpha < 1$

Hence, the correct option is (C).

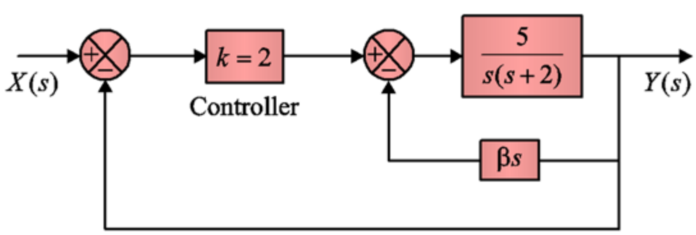
Q.28 For the control system shown below, the value of β to make the damping ratio ξ of the system equal to 0.5 is



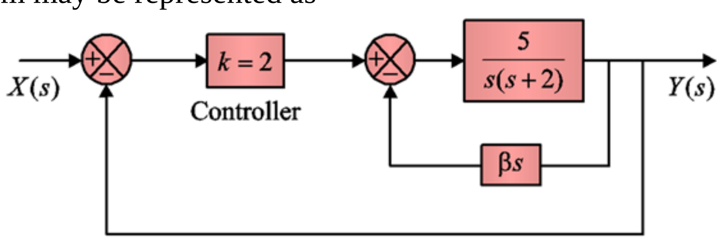
(A) 1.6 (B) 0.232 (C) 0.5 (D) 2.5

Ans. **(B)**

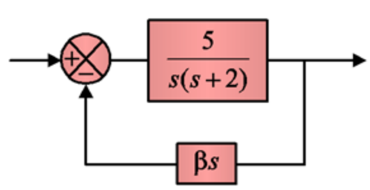
Sol.



The equivalent system may be represented as



Overall transfer function of



Will be

$$G_1(s) = \frac{\frac{5}{s(s+2)}}{1+\beta s.\frac{5}{s(s+2)}} = \frac{\frac{5}{s(s+2)}}{1+\frac{5\beta}{s+2}}$$

$$= \frac{5}{s^2+2s+5\beta s}$$

When $K = 2$ gain gets doubled in the forward path.

$$G_2(s) = \frac{10}{s^2+(2+5\beta)s}$$

Now, the overall transfer function becomes

$$T(s) = \frac{G_2}{1+G_2} = \frac{\frac{10}{s^2+(2+5\beta)s}}{1+\frac{10}{s^2+(2+5\beta)s}}$$

$$T(s) = \frac{10}{s^2+(2+5\beta)s+10}$$

Comparing with the standard characteristic equation

$$s^2 + 2\xi\omega_n s + \omega_n^2$$

We get, $\omega_n^2 = 10$

$\Rightarrow$ $\omega_n = \sqrt{10}$

Also, $2\xi\omega_n = 2+5\beta$

Since, we need $\xi = 0.5$ and $\omega_n = \sqrt{10}$

$\Rightarrow$ $\sqrt{10} = 2+5\beta$

$\Rightarrow$ $\boxed{\beta = 0.232}$ **Ans.**

Hence, correct option is (B).

Q.29 A system is characterized by the following state-space equations

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-3 & 1\\ -2 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix} u\,;\, t>0$$

$$y = [1 \quad 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

The state of the system for a unit-step input at $t = 1\text{sec}$ under zero initial conditions is

(A) $x = \begin{bmatrix}0.2\\ 0.831\end{bmatrix}$ (B) $x = \begin{bmatrix}0.831\\ 0.2\end{bmatrix}$ (C) $x = \begin{bmatrix}0.4\\ 1.631\end{bmatrix}$ (D) $x = \begin{bmatrix}1.631\\ 0.4\end{bmatrix}$

Ans. (A)

Sol. Given : $$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-3 & 1\\ -2 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix} u\,;\, t>0 \quad ...(i)$$

$$y = [1 \quad 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix} \quad ...(ii)$$

State equation is given by,

$$\dot{x} = Ax + Bu \quad ...(iii)$$

Output state equation is given by,

$$y = Cx + Du \quad ...(iv)$$

On comparing equation (i), (ii), (iii) and (iv), we get

$$A = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{and} \quad C = [1 \quad 0]$$

The transfer function of state space model is given by,

$$T(s) = C[sI - A]^{-1} B$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} s+3 & -1 \\ 2 & s \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{\text{Adj}[sI - A]}{|sI - A|}$$

$$\text{Adj}[sI - A] = \begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix}$$

$$|sI - A| = s(s+3) + 2 = s^2 + 3s + 2$$

$$[sI - A]^{-1} = \begin{bmatrix} \frac{s}{s^2+3s+2} & \frac{1}{s^2+3s+2} \\ \frac{-2}{s^2+3s+2} & \frac{s+3}{s^2+3s+2} \end{bmatrix}$$

$$T(s) = [1 \quad 0] \begin{bmatrix} \frac{s}{s^2+3s+2} & \frac{1}{s^2+3s+2} \\ \frac{-2}{s^2+3s+2} & \frac{s+3}{s^2+3s+2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$T(s) = [1 \quad 0] \begin{bmatrix} \frac{1}{s^2+3s+2} \\ \frac{s+3}{s^2+3s+2} \end{bmatrix}$$

$$T(s) = \frac{1}{s^2+3s+2} = \frac{1}{(s+1)(s+2)}$$

State transition matrix is given by,

$$\phi(t) = e^{At} = L^{-1}[sI - A]^{-1}$$

$$[sI - A]^{-1} = \begin{bmatrix} \frac{s}{s^2+3s+2} & \frac{1}{s^2+3s+2} \\ \frac{-2}{s^2+3s+2} & \frac{s+3}{s^2+3s+2} \end{bmatrix}$$

$$L^{-1}[sI - A]^{-1} = L^{-1} \begin{bmatrix} \frac{s}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s+3}{(s+1)(s+2)} \end{bmatrix}$$

Apply partial fraction, we get

$$L^{-1}[sI-A]^{-1} = L^{-1}\begin{bmatrix} \frac{-1}{(s+1)}+\frac{2}{(s+2)} & \frac{1}{(s+1)}-\frac{1}{(s+2)} \\ \frac{-2}{(s+1)}+\frac{2}{(s+2)} & \frac{2}{(s+1)}-\frac{2}{(s+2)} \end{bmatrix}$$

Taking inverse Laplace transform, we get

$$\phi(t) = e^{At} = \begin{bmatrix} -e^{t}+2e^{-2t} & e^{-t}-e^{-2t} \\ -2e^{-t}+2e^{-2t} & 2e^{-t}-e^{-2t} \end{bmatrix}$$

State equation for a unit-step input

$$x(t) = ZIR + ZSR$$

$$x(t) = \phi(t)x(0) + L^{-1}[\phi(s)BU(s)] \qquad (\because x(0)=0,\ ZIR=0)$$

Calculation of ZSR :

$$x(t) = L^{-1}\left\{\begin{bmatrix} \frac{s}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s+3}{(s+1)(s+2)} \end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}\frac{1}{s}\end{bmatrix}\right\}$$

$$x(t) = L^{-1}\left\{\begin{bmatrix} \frac{s}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s+3}{(s+1)(s+2)} \end{bmatrix}\begin{bmatrix}0\\ \frac{1}{s}\end{bmatrix}\right\}$$

$$x(t) = L^{-1}\begin{bmatrix} \frac{1}{s(s+1)(s+2)} \\ \frac{s+3}{s(s+1)(s+2)} \end{bmatrix}$$

Applying partial fraction, we get

$$x(t) = L^{-1}\begin{bmatrix} \frac{1}{2s}-\frac{1}{s+1}+\frac{1}{2(s+2)} \\ \frac{3}{2s}-\frac{2}{s+1}+\frac{1}{2(s+2)} \end{bmatrix}$$

$$\therefore \qquad x(t) = \begin{bmatrix} \frac{1}{2}u(t)-e^{-t}u(t)+\frac{1}{2}e^{-2t}u(t) \\ \frac{3}{2}u(t)-2e^{-t}u(t)+\frac{1}{2}e^{-2t}u(t) \end{bmatrix}$$

At $t = 1\text{sec}$

$$x = \begin{bmatrix} 0.2 \\ 0.831 \end{bmatrix}$$ **Ans.**

Hence, the correct option is (A).

Q.30 The response of a second-order under damped system to a unit step input is known to be,

$$c(t) = 1 - 1.414e^{-\alpha t}\sin(7.5t + \theta)$$

The value of α is _________.

Ans. **7.3 to 7.7 (7.5)**

Sol. The step response is given as,

$$c(t) = 1 - 1.414e^{-\alpha t}\sin(7.5t + \theta)$$

Comparing this equation with standard form,

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_n t + \cos^{-1}\xi)$$

$$\frac{1}{\sqrt{1-\xi^2}} = 1.414 \text{ so that } \xi = 0.707$$

And $\theta = \cos^{-1}\xi = \cos^{-1}0.707 = \frac{\pi}{4}$

$$\omega_d = 7.5 \text{ rad/s}$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}} = 7.5 \times 1.414 = 10.61 \text{ rad/s}$$

$$\alpha = \xi\omega_n = 0.707 \times 10.61 = 7.5$$ **Ans.**

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