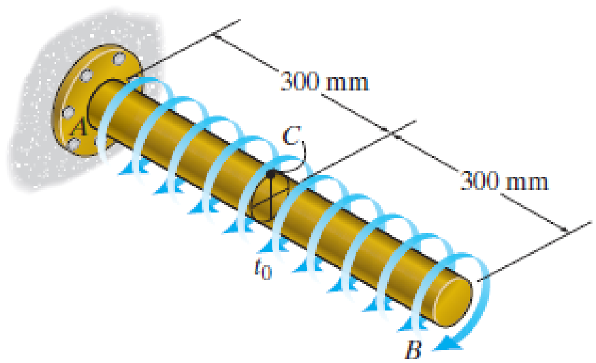
Q.1 If the rod is subjected to a uniform distributed torque of $t_0 = 1.5\,\text{kN-m/m}$, determine the rod's minimum required diameter d if the material has an allowable shear stress of $\tau_{allow} = 75\,\text{MPa}$.



Ans. $d = 39.4$ mm

Sol.

Internal loadings : The maximum internal torque developed in the shaft, which occurs at A, is shown in figure.

Allowable shear stress : The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{32}d^4$.

We have

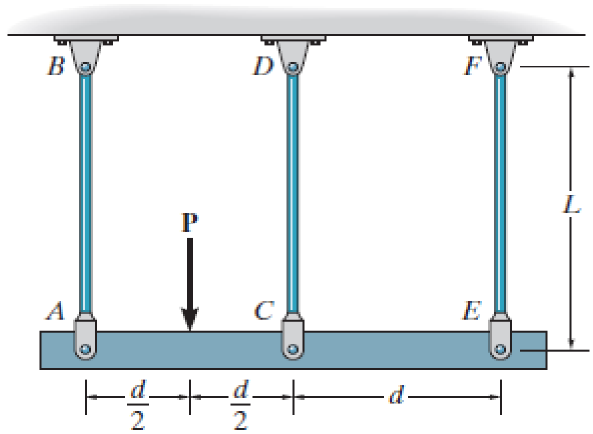
$$\tau_{allow} = \frac{T_{max}c}{J},$$

$$75(10^6) = \frac{1.5(10^3)(0.6)\left(\frac{d}{2}\right)}{\frac{\pi}{32}d^4}$$

$$d = 0.03939\text{ m} = 39.4\text{ mm}$$

Ans.

Q.2 The three suspender bars are made of the same material and have equal cross-sectional areas A. Determine the average normal stress in each bar if the rigid beam ACE is subjected to the force P.



Ans. $\sigma_{AB}=\dfrac{7P}{12A},\ \sigma_{CD}=\dfrac{7P}{3A},\ \sigma_{EF}=\dfrac{P}{12A}$

Sol.

$$\circlearrowright +\sum M_A=0,$$

$$F_{CD}(d)=F_{EF}(2d)-P\left(\frac{d}{2}\right)=0$$

$$F_{CD}+2F_{EF}=\frac{P}{2} \quad \text{....(i)}$$

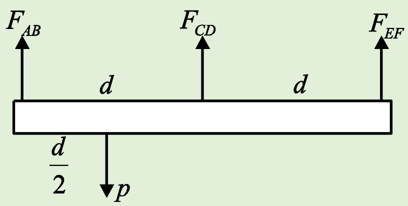
$$+\uparrow\sum F_y=0,\ \ F_{AB}+F_{CD}+F_{EF}-P=0 \quad \text{....(ii)}$$

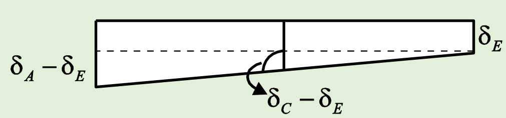
$$\frac{\delta_C-\delta_E}{d}=\frac{\delta_A-\delta_E}{2d}$$

$$2\delta_C=\delta_A+\delta_E$$

$$\frac{2F_{CD}L}{AE}=\frac{F_{AB}L}{AE}+\frac{F_{EF}L}{AE}$$

$$2F_{CD}-F_{AB}-F_{EF}=0 \quad \text{....(iii)}$$





Solving equation (i), (ii) and (iii) yields

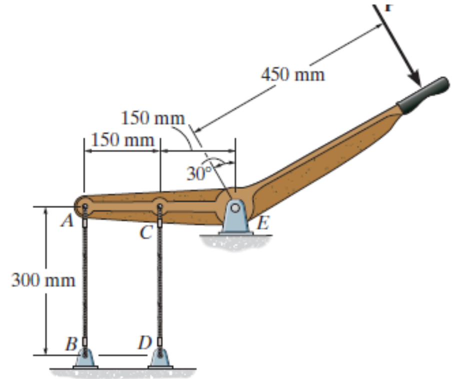
$$F_{AB}=\frac{7P}{12},\ \ F_{CD}=\frac{P}{3},\ \ F_{EF}=\frac{P}{12}$$

$$\sigma_{AB}=\frac{7P}{12A}$$ **Ans.**

$$\sigma_{CD}=\frac{P}{3A}$$ **Ans.**

$$\sigma_{EF} = \frac{P}{12A}$$ **Ans.**

Q.3 The rigid lever arm is supported by two A-36 steel wires having the same diameter of 4 mm. Determine the smallest force P that will cause (a) only one of the wires to yield: (b) both wires to yield. Consider A-36 steel as an elastic perfectly plastic material.



Ans. (a) $P = 2.62$ kN (b) $P = 3.14$ kN

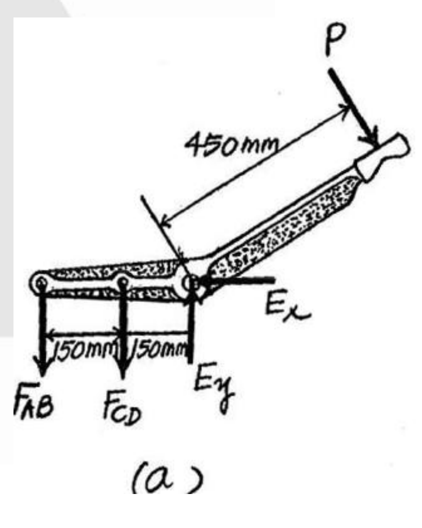
Sol.

Equation of Equilibrium : Referring to the free-body diagram of the lever arm shown in figure (a),

$$\circlearrowleft + \sum M_E = 0, \quad F_{AB}(300) + F_{CD}(150) - P(450) = 0$$

$$2F_{AB} + F_{CD} = 3P \qquad \text{....(i)}$$

(a) Elastic Analysis : The compatibility equation can be written by referring to the geometry of fig.(b)



$$\delta_{AB} = \left(\frac{300}{150}\right)\delta_{CD}$$

$$\delta_{AB} = 2\delta_{CD}$$

$$\frac{F_{AB}L}{AE} = 2\left(\frac{F_{CD}L}{AE}\right)$$

$$F_{CD} = \frac{1}{2}F_{AB} \qquad \text{... (ii)}$$

Assuming that wire AB is about to yield first,

$$F_{AB} = (\sigma\gamma)_{st} A_{AB} = 250(10^6)\left[\frac{\pi}{4}(0.004)^2\right] = 3141.59 \text{ N}$$

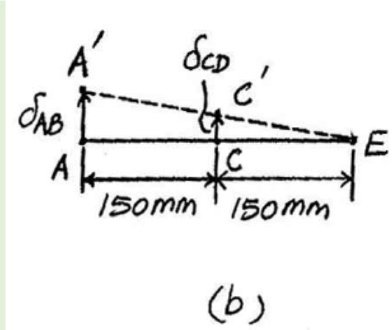
From equation (ii),

$$F_{CD} = \frac{1}{2}(3141.59) = 1570.80 \text{ N}$$

Substitution the result of F_{AB} and F_{CD} into equation (i),

$$P = 2618.00 \text{ N} = 2.62 \text{ kN}$$

(b) Plastic Analysis : Since both wires AB and CD are required to yield,



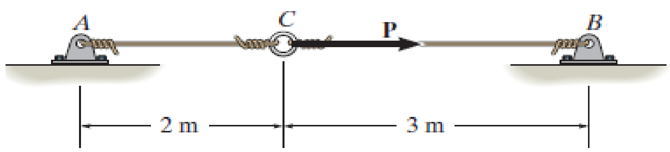
$$F_{AB} = F_{CD} - (\sigma\gamma)_{st} A 250(10^6)\left[\frac{\pi}{4}(0.004^2)\right] = 3141.59 \text{ N}$$

Substituting this result into equation (i),

$P = 3141.59 \text{ N} = 3.14 \text{ kN}$ **Ans.**

Q.4 Two steel wires, each having a cross-sectional area of $\mathbf{2\,mm^2}$ are tied to a ring at *C*, and then stretched and tied between the two pins *A* and *B*. The initial tension in the wires is 50 N. If a horizontal force *P* is applied to the ring, determine the force in each wire if *P* = 20 N. What is the smallest force *P* that must be applied to the ring to reduce the force in wire *CB* to zero?

Take $\sigma_Y = \mathbf{300\ MPa}$. $E_{st} = \mathbf{200\ GPa}$.



Ans. $F_{AC} = 62\,\text{N},\ F_{BC} = 42\,\text{N},\ P = 125\,\text{N}$

Sol.

Equilibrium :

$$\xrightarrow{+} \sum Fx = 0,\ 20 + (50 - P_2) - (50 + P_1) = 0$$

$$P_1 + P_2 = 20 \qquad \ldots \text{(i)}$$

Compatibility condition :

$$\delta_C = \frac{P_1(2)}{AE} = \frac{P_2(3)}{AE}$$

$$P_1 = 1.5\ P_2 \qquad \ldots \text{(ii)}$$

Solving equation (i) and (ii) yields :

$$P_1 = 12 \text{ N},\ P_2 = 8 \text{ N}$$

$$F_{AC} = 50 + 12 = 62 \text{ N}$$

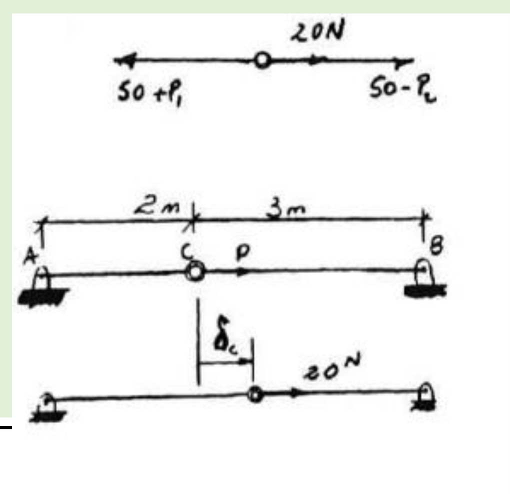
$$F_{BC} = 50 - 8 = 42 \text{ N}$$

For $F_{CB} = 0,\ 50 - P_2 = 0$

$$P_2 = 50 \text{ N}$$

$$P_1 = 1.5(50) = 75 \text{ N}$$

$$P = 75 + 50 = 125 \text{ N}$$

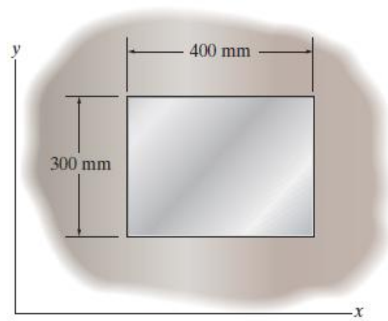


$$F_{At} = 50 + 75 = 125 \text{ N}$$

$$\sigma_{At} = \frac{125}{2(10^{-6})} = 62.5 \text{ MPa}$$

$$62.5 \text{ MPa} < \sigma_Y$$

Q.5 The 6061-T6 aluminum alloy plate fits snugly into the rigid constraint. Determine the normal stresses σ_x and σ_y developed in the plate if the temperature is increased by $\Delta T = 50^0\text{C}$. To solve, add the thermal strain $\alpha\Delta T$ to the equations for Hooke's Law. Coefficient of thermal expansion= $\alpha = 24\times10^{-6}$ and E= $68 GPa$



Ans. $\sigma_x = \sigma_y = -127.2 \text{ MPa} = 127 \text{ MPa (C)}$

Sol.

Generalized Hooke's Law : Since the sides of the aluminum plate are confined in the rigid constraint along the x and y directions, $\in_x = \in_y = 0$. However, the plate is allowed to have free expansion along the *z* direction. Thus, $\sigma_z = 0$. With the additional thermal strain term, we have

$$\in_x = \frac{1}{E}[\sigma_x - v(\sigma_y + \sigma_z)] + \alpha\Delta T$$

$$0 = \frac{1}{68.9(10^9)}[\sigma_x - 0.35(\sigma_y + 0)] + 24(10^{-6})(50)$$

$$\sigma_x - 0.35\sigma_y - 82.68(10^6) \qquad \dots \text{(i)}$$

$$\in_y = \frac{1}{E}[\sigma_y - v(\sigma_x + \sigma_z)] + \alpha\Delta T$$

$$0 = \frac{1}{68.9(10^9)}[\sigma_y - 0.35(\sigma_x + 0)] + 24(10^{-6})(50)$$

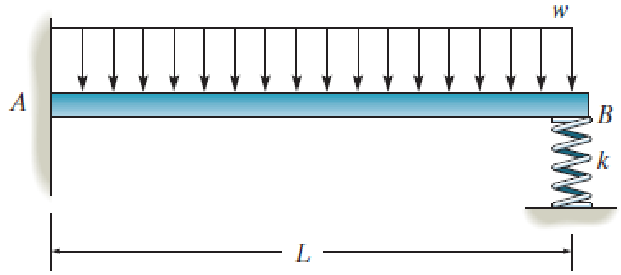
$$\sigma_y - 0.35\sigma_x = -82.68(10^6) \qquad \dots \text{(ii)}$$

Solving equation (i) and (ii),

$$\sigma_x = \sigma_y = -127.2 \text{ MPa} = 127 \text{ MPa(C)}$$ **Ans.**

Since $\sigma_x = \sigma_y$ and $\sigma_y < \sigma_Y$, the above results are valid.

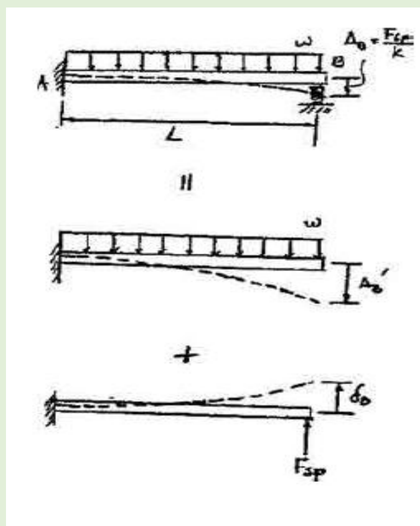
Q.6 Determine the force in the spring. *EI* is constants



Ans. $F_{sp} = \dfrac{3kwL^4}{24EI + 8kL^3}$

Sol.

$$\Delta_B' = \frac{wL^4}{8EI}, \qquad \delta_B = \frac{F_{sp}L^3}{3EI}$$



By superposition :

$$+\downarrow\ \Delta_B = \Delta_B' - \delta_B$$

$$\frac{F_{sp}}{k} = \frac{wL^4}{8EI} - \frac{F_{sp}L^3}{3EI}$$

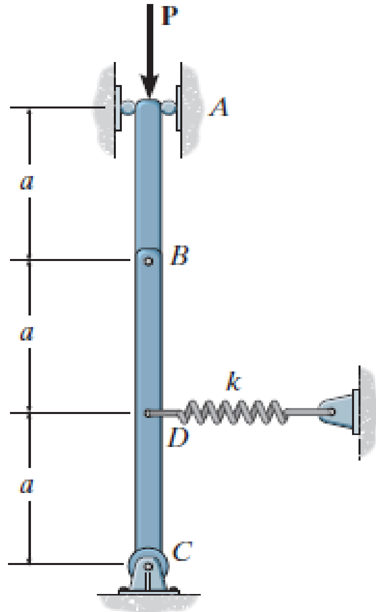
$$\frac{24EIF_{sp}}{k} = 3wL^4 - 8F_{sp}L^3$$

$$\frac{24EIF_{sp}}{k} + 8F_{sp}L^3 = 3wL^4$$

$$F_{sp}\left[\frac{24EI + 8kL^3}{k}\right] = 3wL^4$$

$$F_{sp} = \frac{3kwL^4}{24EI + 8kL^3}$$

Q.7 Rigid bars *AB* and *BC* are pin connected at *B*. If the spring at *D* has a stiffness *k*, determine the critical load P_{cr} for the system.



Ans. $P_{cr} = \dfrac{ka}{6}$

Sol.

Equilibrium : The disturbing force F can be related P by considering the equilibrium of joint A and then the equilibrium of member BC,

Joint A (fig. b)

$$+\uparrow \sum F_y = 0,\ F_{AB}\cos\phi - P = 0,\ F_{AB} = \frac{P}{\cos\phi}$$

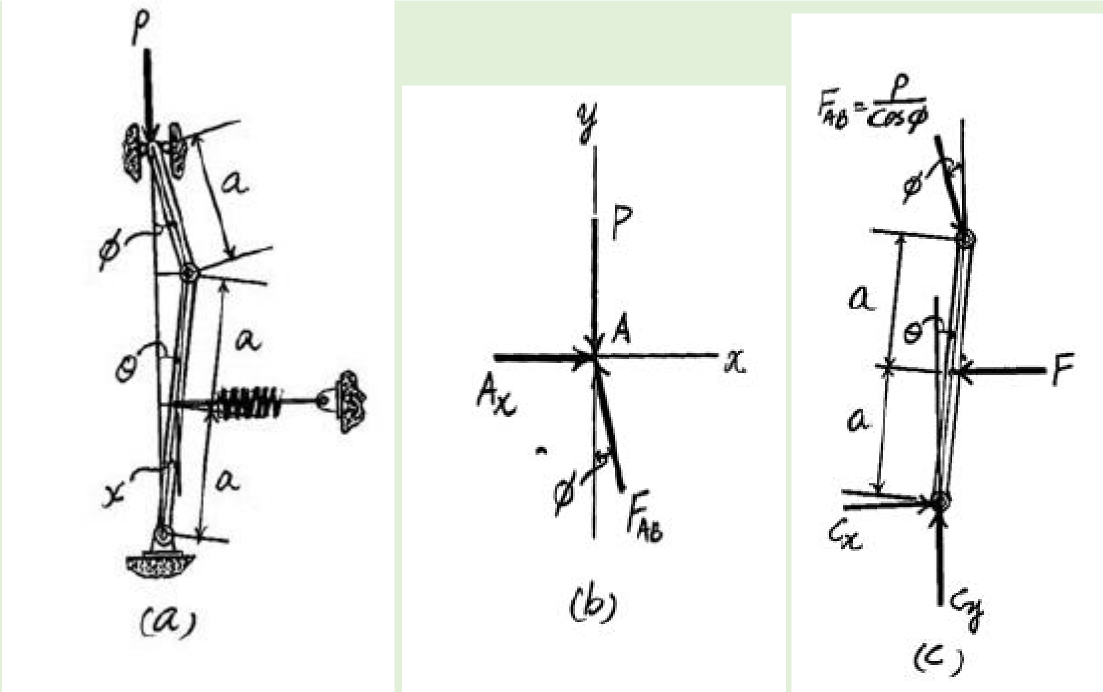
Member BC (fig. c)

$$\sum M_C = 0,\ F(a\cos\theta) - \frac{P}{\cos\phi}\cos\phi(2a\sin\theta) - \frac{P}{\cos\phi}\sin(2a\cos\theta) = 0$$

$$F = 2P(\tan\theta + \tan\phi)$$

Since θ and ϕ are small, $\tan\theta \cong \theta$ and $\tan\phi \cong \phi$. Thus,

$$F = 2P(\theta + \phi) \qquad \ldots \text{(i)}$$



Also, from the geometry shown in fig. a,

$$2a\theta = a\phi \qquad \phi = 2\theta$$

Thus, equation (i) becomes

$$F = 2P(\theta + 2\theta) = 6P\theta$$

Spring force : The restoring spring force F_{sp} can be determined using the spring formula, $F_{sp} = kx$, where $x = a\theta$, fig.(a), thus,

$$F_{sp} = kx = ka\theta$$

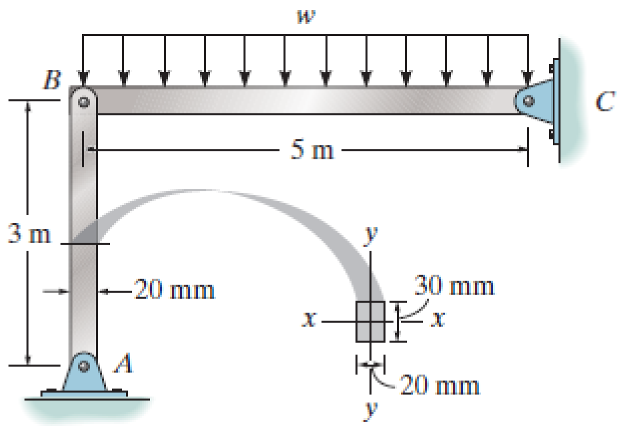
Critical buckling load : When the mechanism is on the verge of buckling the disturbing force F must be equal to the restoring spring force F_{sp}.

$$F = F_{sp}$$

$$6P_{cr}\theta = ka\theta$$

$$P_{cr} = \frac{ka}{6}$$ Ans.

Q.8 The steel bar *AB* has a rectangular cross section. If it is pin connected at its ends, determine the maximum allowable intensity *w* of the distributed load that can be applied to *BC* without causing bar *AB* to buckle. Use a factor of safety with respect to buckling of **1.5,** $E_{st} = $ **200 GPa,** $\sigma_Y = $ **360 MPa** .



Ans. $P_{cr} = 4.39\,\text{kN}$

Sol.

Buckling load :

$$P_{cr} = F_{AB}(F.S.) = 2.5w(1.5) = 3.75\ w$$

$$I = \frac{1}{12}(0.03)(0.02)^3 = 20(10^{-9})\ \text{m}^4$$

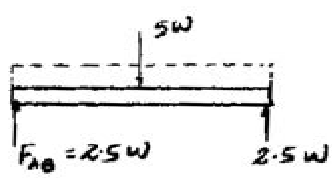
$$K = 1.0$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$3.75w = \frac{\pi^2(200)(10^9)(20)(10^{-9})}{[(1.0)(3)]^2}$$

$$w = 1170\ \text{N/m} = 1.17\ \text{kN/m}$$

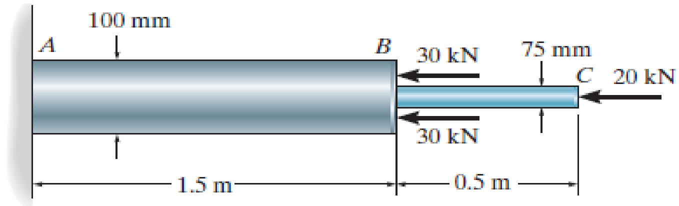
$$P_{cr} = 4.39\ \text{kN}$$



Check :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{4.39(10^3)}{0.02(0.03)} = 7.31\ \text{MPa} < \sigma_Y$$

Q.9 Determine the strain energy in the stepped rod assembly. Portion *AB* is steel and *BC* is brass. $E_{br} = 101$ **GPa,** $E_{st} = 200$ **MPa,** $(\sigma_Y)_{br} = 410$ **MPa,** $(\sigma_Y)_{st} = 250$ **MPa.**



Ans. $U_i = 3.28\text{ J}$

Sol.

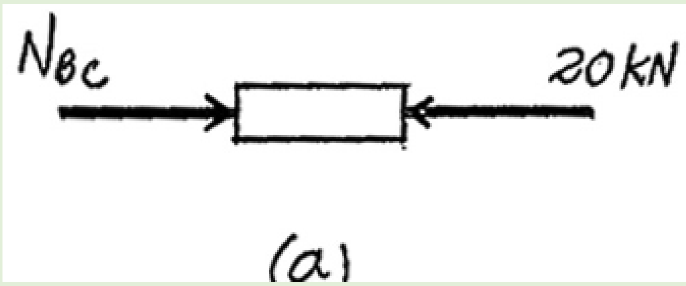
Referring to the FBDs of cut segments in fig. (a) and (b),

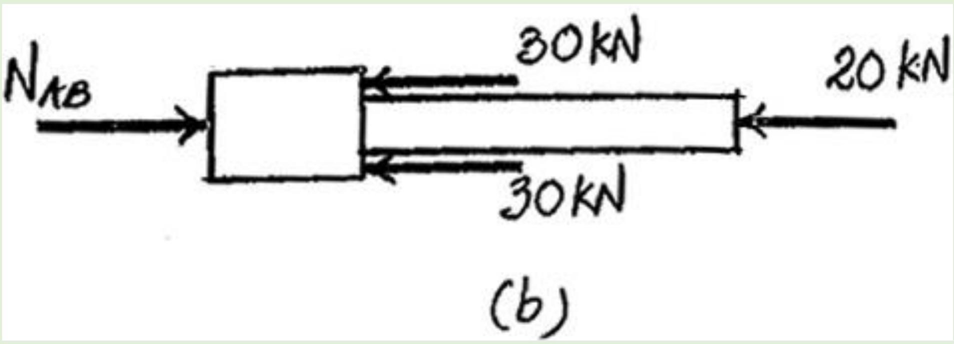
$$\overset{+}{\rightarrow}\sum F_x = 0,\; N_{BC} - 20 = 0,\; N_{BC} = 20\text{ kN}$$

$$\overset{+}{\rightarrow}\sum F_x = 0,\; N_{BC} - 30 - 30 - 20 = 0,\; N_{AB} = 80\text{ kN}$$

The cross-sectional area of segments *AB* and *BC* are $A_{AB} = \frac{\pi}{4}(0.1^2) = 2.5(10^{-3})\pi\,\text{m}^2$

and $A_{AB} = \frac{\pi}{4}(0.075^2) = 1.40625(10^{-3})\pi\,\text{m}^2$





$$(U_i)_a = \sum\frac{N^2L}{2AE} = \frac{N_{AB}^2\,L_{AB}}{2A_{AB}E_{st}} + \frac{N_{BC}^2\,L_{BC}}{2A_{BC}E_{br}}$$

$$= \frac{[80(10^3)]^2(1.5)}{2[2.5(10^{-3})\pi][200(10^9)]} + \frac{[20(10^3)]^2(0.5)}{2[1.40625(10^{-3})\pi][101(10^9)]}$$

$$= 3.28\text{ J}$$

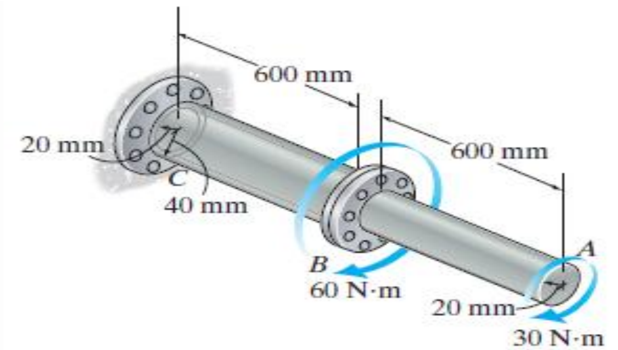
Ans.

This result is valid only if $\sigma < \sigma_y$

$$\sigma_{AB} = \frac{N_{AB}}{A_{AB}} = \frac{80(10^3)}{2.5(10^{-3})\pi} = 10.19(10^6)\text{ Pa} = 10.19\text{ MPa} < (\sigma_y)_{st} = 250\text{ MPa}$$

$$\sigma_{BC} = \frac{N_{BC}}{A_{BC}} = \frac{20(10^3)}{1.40625(10^{-3})\pi} = 4.527(10^6)\text{ Pa} = 4.527\text{ MPa} < (\sigma_y)_{br} = 410\text{ MPa}$$

Q.10 The shaft assembly is fixed at *C*. The hollow segment *BC* has ail inner radius of 20 mm and outer radius of 40 mm, while the solid segment *AB* has a radius of 20 mm Determine the torsional strain energy stored in the shaft. The shaft is made of 2014-T6 aluminum alloy. The coupling at B is rigid. Where G= 27 GPa.



Ans. $U_i = 0.0638\text{ J}$

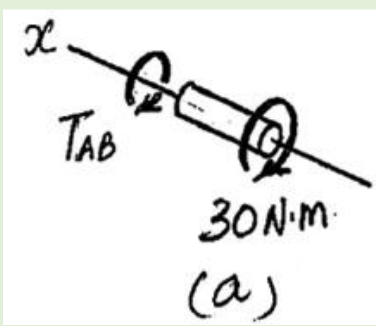
Sol.

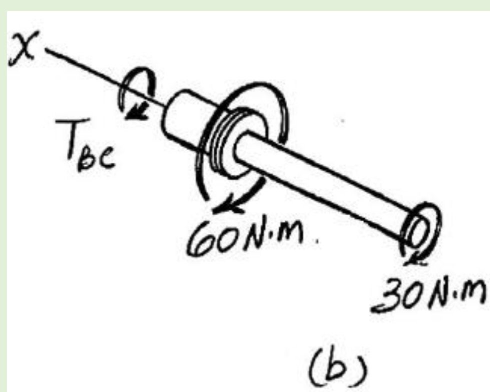
Internal Torque : Referring to the free body diagram of segment *AB*, fig. a,

$\sum M_x = 0,\ T_{AB} + 30 = 0,\ T_{AB} = -30\text{ Nm}$

Referring to the free body diagram of segment *BC*, fig. b,

$\sum M_x = 0,\ T_{BC} + 30 + 60 = 0,\ T_{BC} = -90\text{ Nm}$





Torsional Strain Energy : Here, $J_{AB} = \frac{\pi}{2}(0.02^4) = 80(10^{-9})\pi\text{ m}^4$ and

$$J_{BC} = \frac{\pi}{2}(0.04^4 - 0.02^4) = 1200(10^{-9})\pi\text{ m}^4$$

$$(U_i)_t = \sum \frac{T^2 L}{2GJ} = \frac{T_{AB}^{\ 2}\ L_{AB}}{2GJ_{AB}} + \frac{T_{BC}^{\ 2}\ L_{BC}}{2GJ_{BC}}$$

$$= \frac{(-30)^2(0.6)}{2[27(10^9)][80(10^{-9})\pi]} + \frac{(-90)^2(0.6)}{2[27(10^9)][1200(10^{-9})\pi]}$$

$$= 0.06379 \text{ J} = 0.0638 \text{ J}$$

Ans.