Classroom Test : 2016 - 17

Signal & Systems (EC /EE/EEE/ IN)

Duration : 90 Minutes **Maximum Marks : 60**

Q.1 to Q.10 carry one mark each

Q.1 Consider a continuous-time system with input $x(t)$ and output $y(t)$ given by

$$y(t) = x(t)\cos(t)$$

This system is

(A) linear and time-invariant
(B) non-linear and time-invariant
(C) linear and time-varying
(D) non-linear and time-varying

Sol. **Method 1 :**

$$y(t) = x(t)\cos(t)$$

The given system is of the form

$$y = mx \text{ (Equation of straight line)}$$

$\therefore$ It is linear

$\because$ There is an extra term of "t" in $\cos(t)$

$\therefore$ $y(t)$ is time-variant

Hence, the correct option is (C).

Method 2 :

A system is said to be linear if it is both additive and homogeneous.

Additivity : Addition of inputs and outputs.

Direct change in input and output,

$$y_1(t) = x_1(t)\cos(t) \quad \text{....(i)}$$

$$y_2(t) = x_2(t)\cos(t) \quad \text{....(ii)}$$

Adding equations (i) and (ii),

$$y_1(t) + y_2(t) = x_1(t)\cos(t) + x_2(t)\cos(t) \quad \text{....(A)}$$

Direct change in system,

$$[y_1(t) + y_2(t)] = [x_1(t) + x_2(t)]\cos(t) \quad \text{....(B)}$$

$\because$ (A) = (B) $\Rightarrow$ System is additive.

Homogenity : Multiplication by a constant.

Direct change in input and output,

$$k\,y(t) = k\,x(t)\cos(t) \quad \text{....(C)}$$

Direct change in system,

$$k\,y(t)=k[x(t)\cos(t)] \quad \text{....(D)}$$

∵ (C) = (D) ⇒ System is homogeneous.

∴ System is linear.

Time-invariant : A system is said to be time-invariant if,

$$y(t-t_0)=T[x(t-t_0)]$$

Direct change in input and output,

$$y(t-t_0)=x(t-t_0)\cos(t) \quad \text{....(E)}$$

Direct change in system,

$$y(t-t_0)=x(t-t_0)\cos(t-t_0) \quad \text{....(F)}$$

∵ (E) ≠ (F) ⇒ System is time-variant

∴ System is linear and time-variant.

Hence, the correct option is (C).

Ans. (C)

Q.2 The initial value of $X(s)=\dfrac{e^{-2s}(s+1)}{(s+2)}$ is

(A) 0 (B) 1

(C) 2 (D) Cannot be applied

Sol.

$$X(s)=\frac{e^{-2s}(s+1)}{(s+2)}$$

There is an exponential term so it is best to prefer time domain approach.
Exponential term provides shifting which we will easily observe in time domain.

$$X(s)=\frac{e^{-2s}(s+1+1-1)}{(s+2)}=\frac{e^{-2s}(s+2-1)}{(s+2)}$$

$$X(s)=e^{-2s}-\frac{e^{-2s}}{(s+2)}$$

Taking inverse Laplace transform on both sides, we get

$$x(t)=\delta(t-2)-e^{-2(t-2)}u(t-2)$$

Initial value of $x(t)$ is given by,

$$x(0)=\lim_{t\to 0}x(t)=\delta(-2)-e^{0}\,u(-2)$$

$$x(0)=0-0=0$$

Hence, the correct option is (A).

Ans. (A)

Q.3 The fundamental D.T. period of signal $g[n]=\text{Re}\left[e^{j\pi n}+e^{-j\frac{\pi n}{3}}\right]$ is N_0 = ______________.

Sol.

$$g(n)=\text{Re}\left[e^{j\pi n}+e^{-j\frac{n\pi}{3}}\right]$$

$$g(n)=\text{Re}\left[\cos n\pi+j\sin n\pi+\cos\frac{n\pi}{3}-j\sin\frac{n\pi}{3}\right]$$

$$g(n)=\cos n\pi+\cos\frac{n\pi}{3}$$

A discrete time signal is said to be periodic if $\frac{m}{N_0}$ is a rational number i.e. ratio of integers.

Let $g_1(n) = \cos n\pi \Rightarrow \omega_1 = \pi$ rad/sec

and $g_2(n) = \cos\frac{n\pi}{3} \Rightarrow \omega_2 = \frac{\pi}{3}$ rad/sec

$$\frac{m}{N_1} = \frac{\omega_1}{2\pi} = \frac{\pi}{2\pi} = \frac{1}{2} \Rightarrow N_1 = 2$$

$$\frac{m}{N_2} = \frac{\omega_2}{2\pi} = \frac{\pi}{3\times 2\pi} = \frac{1}{6} \Rightarrow N_2 = 6$$

Fundamental time period N_0 is given by,

$$N_0 = \text{L.C.M.}(N_1, N_2) = \text{L.C.M.}(2, 6)$$

$$N_0 = 6$$

Hence, the correct answer is 6.

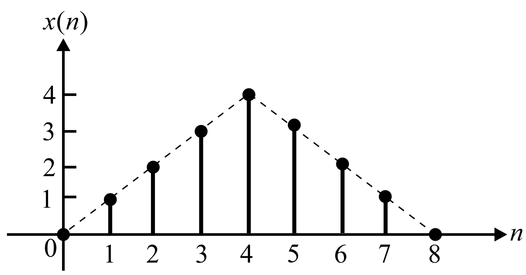
Ans. (6)

Q.4 The energy of the signal $x[n] = r[n] - 2r[n-4] + r[n-8]$ is

(A) ∞ (B) $\frac{11}{4}$

(C) 11 (D) 44

Sol. $x(n) = r(n) - 2r(n-4) + r(n-8)$



Energy of discrete time signal $= \sum_{n=-\infty}^{\infty} x^2(n)$

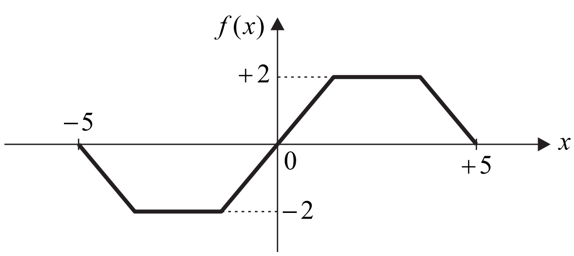
$$E = 0^2 + 1^2 + 2^2 + 3^2 + 4^2 + 3^2 + 2^2 + 1^2 + 0^2$$

$$E = 44$$

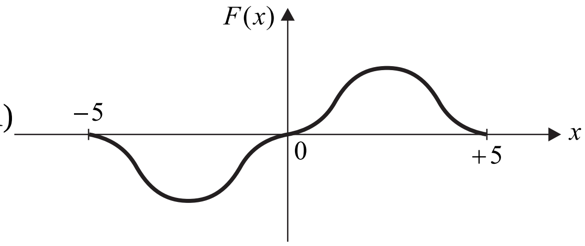
Hence, the correct option is (D).

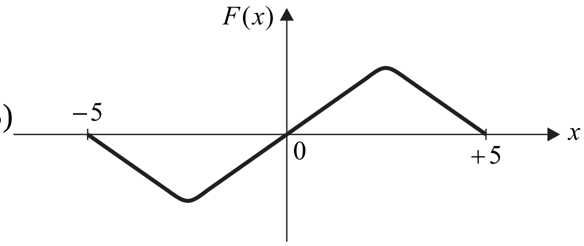
Ans. (D)

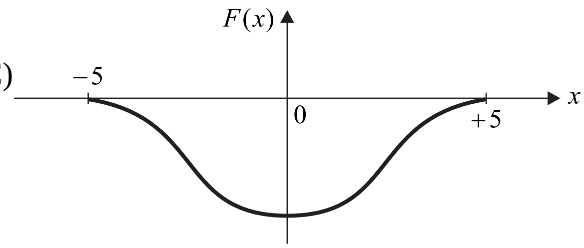
Q.5 Consider the plot of $f(x)$ versus x as shown below.

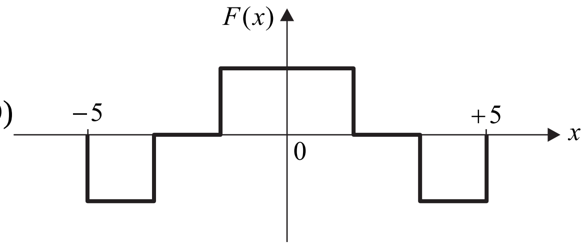


Suppose $F(x)=\int_{-5}^{x} f(y)\,dy$. Which one of the following is a graph of $F(x)$?

(A)


(B)


(C)


(D)


Sol. **Method 1 :**

Given : $F(x)=\int_{-5}^{x} f(y)dy$

Then $F'(x)=f(x)$ which is a density function.

From the given figure,

$F'(x)=f(x)<0$, when $x<0$

Hence $F(x)$ is decreasing function for $x<0$

$F'(x)=f(x)>0$, when $x>0$

Hence $F(x)$ is increasing function for $x>0$

Hence, the correct option is (C).

Method 2 :

Integration of an odd signal is even signal.

Only option (C) and (D) are even.

$$\text{Step} \xrightarrow{\text{Integration}} \text{Ramp}$$

$$\text{Ramp} \xrightarrow{\text{Integration}} \text{Parabola}$$

Hence, the correct option is (C).

Ans. **(C)**

Q.6 The energy of the signal $x[n]=\begin{cases} A & -N_0 \le n \le N_0 \\ 0 & \text{Elsewhere} \end{cases}$ is

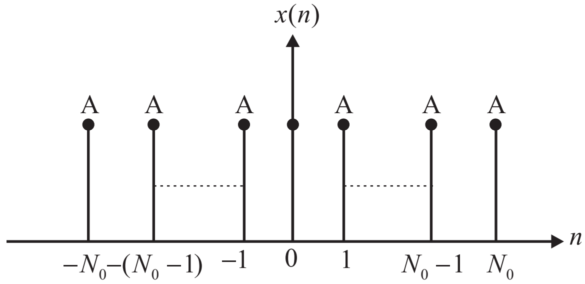
(A) $2N_0A$

(B) $2N_0A^2$

(C) $(2N_0+1)A^2$

(D) $(2N_0+1)A$

Sol. $x[n]=\begin{cases} A, & -N_0 \le n \le N_0 \\ 0, & \text{elsewhere} \end{cases}$



Energy of $x(n)$ is,

$$E = \sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-N_0}^{N_0} A^2$$

$$= (2N_0 + 1)A^2$$

Ans. (C)

Q.7 The value of $\sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n$ is ________________.

Sol. Method 1 : Using concept of infinite convergence series :

$$\sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n = \sum_{n=0}^{\infty} n(\alpha)^n$$

$$\sum_{n=0}^{\infty} n(\alpha)^n = \frac{\alpha}{(1-\alpha)^2}; \quad |\alpha| < 1$$

$\therefore$
$$S = \frac{\frac{1}{2}}{\left(1-\frac{1}{2}\right)^2} = 2$$

Method 2 : Using concept of Z-transform :

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$X(z)\big|_{z=1} = X(1) = \sum_{n=-\infty}^{\infty} x(n) = \text{area of action}$$

$$x(n) = n\left(\frac{1}{2}\right)^n u(n) \qquad \left[\because na^n u(n) \xleftrightarrow{\text{Z.T.}} \frac{az}{(z-a)^2}\right]$$

$$X(z) = \frac{\frac{1}{2}z}{\left(z-\frac{1}{2}\right)^2}$$

$$X(1) = \frac{\frac{1}{2}}{\left(1-\frac{1}{2}\right)^2} = 2$$

Ans. 2

Q.8 If $X(s)$, the Laplace transform of signal $x(t)$ is given by $X(s)=\dfrac{(s+2)}{(s+1)(s+3)^2}$, then the value of $x(t)$ as $t\to\infty$ is________.

Sol. Final value theorem is valid for stable signal

$X(s)$ has three poles on left hand side of s-plane. So it is stable.

Using final value theorem,

$$\lim_{t\to\infty} x(t)=\lim_{s\to 0} sX(s)=\lim_{s\to 0}\frac{s(s+2)}{(s+1)(s+3)^2}=0$$

Ans. 0

Q.9 Two sequences $x_1[n]$ and $x_2[n]$ have the same energy. Suppose $x_1[n]=\alpha\, 0.5^n\, u[n]$, where α is a positive real number and $u[n]$ is the unit step sequence. Assume $x_2[n]=\begin{cases}\sqrt{1.5} & \text{for } n=0,1\\ 0 & \text{otherwise}\end{cases}$. Then the value of α is ______.

Sol.

$$x_1[n]=\alpha.(0.5)^n u(n)$$

$$E[x_1(n)]=\sum_{n=-\infty}^{\infty} x_1^2(n)$$

$$E[x_1(n)]=\sum_{n=-\infty}^{\infty} \alpha^2(0.25)^n u^2(n)$$

$$=\sum_{n=0}^{\infty}\alpha^2(0.25)^n \qquad [u^2(n)=u(n)]$$

$$=\alpha^2\left(1+0.25+0.25^2+........\right)$$

$$=\alpha^2\times\frac{1}{1-0.25} \qquad \text{[sum of infinite G.P. series with } |r|<1,\ S_\infty=\frac{a}{1-r}\text{]}$$

$$=\frac{\alpha^2}{0.75}=\frac{4\alpha^2}{3}$$

$$x_2(n)=\begin{cases}\sqrt{1.5}, & \text{for } n=0,1\\ 0, & \text{otherwise}\end{cases}$$

$$E[x_2(n)]=\sum_{n=-\infty}^{\infty} x_2^2(n)$$

$$=1.5+1.5=3$$

$$E[x_1(n)]=E[x_2(n)] \qquad \text{(Given)}$$

$$\frac{4\alpha^2}{3}=3$$

$$\alpha=\pm 1.5$$

$$\alpha=1.5 \qquad (+\text{ve real no})$$

Ans. (1.5)

Q.10 Let the signal $f(t) = 0$ outside the interval $[T_1, T_2]$, where T_1 and T_2 are finite. Furthermore, $|f(t)| < \infty$.
The region of convergence (ROC) of the signal's bilateral Laplace transform $F(s)$ is

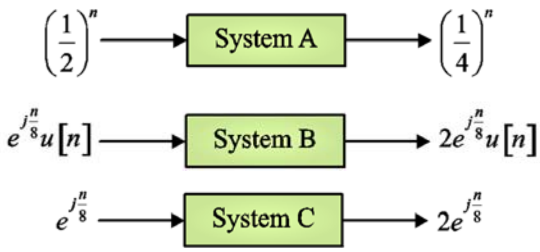
(A) A parallel strip containing the $j\Omega$ axis
(B) A parallel strip not containing the $j\Omega$ axis
(C) The entire s-plane
(D) A half plane containing the $j\Omega$ axis

Sol. For finite duration signal, ROC is entire s-plane.

Ans. **(C)**

Q.11 to Q.35 carry two marks each

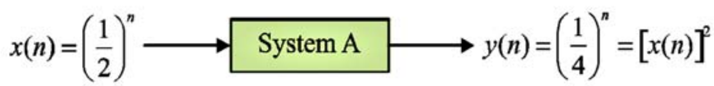
Q.11 Three systems A, B, and C have the inputs and outputs indicated in figure.



The LTI system are

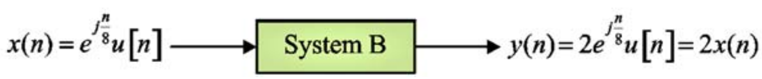
(A) A, B, C
(B) A, B
(C) Only B
(D) B and C

Sol.



$$y[n] = \left(\frac{1}{4}\right)^n = \left(\left(\frac{1}{2}\right)^2\right)^n = \left(\left(\frac{1}{2}\right)^n\right)^2 = [x(n)]^2$$

Hence system A is non-linear, so A is not an LTI system



$$y(n-n_0) = 2x(n-n_0) = 2e^{j\frac{(n-n_0)}{8}}u(n-n_0)$$

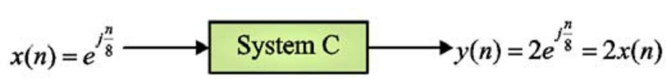
$$T\{x(n-n_0)\} = 2e^{j\frac{(n-n_0)}{8}}u(n-n_0)$$

Since $y(n-n_0) = T\{x(n-n_0)\}$

Hence it is time invariant.

Since it is in the form of $y = mx$, so it is linear system.

Hence system B is an LTI system.



$$y(n-n_0) = 2x(n-n_0) = 2e^{j\frac{(n-n_0)}{8}}$$

$$T\{x(n-no)\} = 2e^{j\frac{(n-n_0)}{8}}$$

Since $y(n-n_0)=T\{x(n-n_0)\}$

Hence it is time invariant.

Since it is in the form of $y=mx$, so it is a linear system.

Hence system C is an LTI system.

Hence the correct option is (D)

Ans. (D)

Q.12 The value of integral $I=\int_{-\infty}^{\infty}[\cos t+\sin t]\delta'(t^3+t^2+t)dt$ is

(A) 0 (B) 1

(C) 2 (D) 3

Sol. $\delta'(x)=\frac{d}{dx}\delta(x)$ [Doublet = first derivative of impulse]

Replace all x by t^3+t^2+t, then

$$\delta'(t^3+t^2+t)=\frac{d\delta(t^3+t^2+t)}{d(t^3+t^2+t)}=\frac{1}{(3t^2+2t+t)}\frac{d}{dt}\delta(t^3+t^2+t)$$

$\because$ $$\delta(t)=\begin{cases}\infty, & t=0\\ 0, & t\neq 0\end{cases}$$

$$\delta(t^3+t^2+t)=\begin{cases}\infty, & t^3+t^2+t=0\\ 0, & t^3+t^2+t\neq 0\end{cases}$$

$$t^3+t^2+t=0 \Rightarrow \quad t=0\,\text{(Real time)}$$

$\therefore$ $$\delta(t^3+t^2+t)=\begin{cases}\infty, & t=0\\ 0, & t\neq 0\end{cases}$$

$$\delta(t^3+t^2+t)=\delta(t)$$

$$\delta(t^3+t^2+t)=\frac{1}{3t^2+2t+1}\frac{d}{dt}\delta(t)=\frac{1}{3t^2+2t+t}$$

$$I=\int_{-\infty}^{\infty}\left[\frac{\cos t+\sin t}{3t^2+2t+1}\right]\delta'(t)dt$$

Use property, $I=\int_{-\infty}^{\infty}\phi(t)\delta'(t)dt=-\phi'(0)=-\left[\frac{d}{dt}\phi(t)\right]_{t=0}$

$\therefore$ $$I=-\left[\frac{(3t^2+2t+1)[-\sin t+\cos t]-[\cos t+\sin t][6t+2]}{(3t^2+2t+1)^2}\right]_{t=0}$$

$$I=-\left[\frac{1\times 1-1\times 2}{1}\right]=-(-1)=1$$

Hence the correct option is (B)

Ans. (B)

Q.13 An LTI system with impulse response $h(t)=u(t)-u(t-2)$ is excited by an input $x(t)=\delta(t+3)+3e^{-0.5t}u(t)$. The response $y(t)$ of the system at $t=2$ is ________.

Sol. Method 1 :

$$y(t)=x(t)\otimes h(t)=h(t)\otimes\delta(t+3)+h(t)\otimes 3e^{-0.5t}u(t)$$

$$=y_1(t)+y_2(t)$$

$$y_1(t)=h(t+3)$$

$$y_1(t)=u(t+3)-u(t+1)$$

$$y_1(2)=0$$

$$y_2(t)=\int_{-\infty}^{\infty}h(\tau)\cdot x(t-\tau)\,d\tau$$

$$y_2(2)=\int_0^2 1\times 3e^{-0.5(2-\tau)}\,d\tau$$

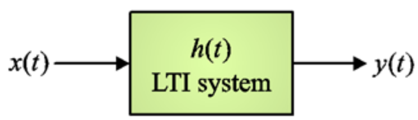
$$y_2(2)=3e^{-1}\int_0^2 e^{-0.5\tau}\,d\tau=\frac{3}{e}\times\left[\frac{e^{0.5\tau}}{0.5}\right]_0^2$$

$$y_2(2)=\frac{3}{e}\times\left[\frac{e^1-1}{0.5}\right]=3.7927$$

Method 2 : Using Laplace transform approach :

$$h(t)=u(t)-u(t-2)$$

and $$x(t)=\delta(t+3)+3e^{-0.5t}u(t)$$



$$y(t)=x(t)\otimes h(t)$$

Taking Laplace transform on both sides, we get

$$Y(s)=X(s)\cdot H(s)$$

$$Y(s)=\left(e^{3s}+\frac{3}{s+0.5}\right)\cdot\left(\frac{1}{s}-\frac{e^{-2s}}{s}\right)$$

$$Y(s)=\frac{e^{3s}}{s}+\frac{3}{s(s+0.5)}-\frac{e^{3s}\cdot e^{-2s}}{s}-\frac{3e^{-2s}}{s(s+0.5)}$$

$$Y(s)=\frac{e^{3s}}{s}+\frac{3}{s(s+0.5)}-\frac{e^{s}}{s}-\frac{3e^{-2s}}{s(s+0.5)}$$

$$Y(s)=\frac{e^{3s}}{s}+\frac{6}{s}-\frac{6}{s+0.5}-\frac{e^{s}}{s}-\frac{6e^{-2s}}{s}+\frac{6e^{-2s}}{s+0.5}$$

Taking inverse Laplace transform on both sides, we get

$$y(t)=u(t+3)+6u(t)-6e^{-0.5t}u(t)-u(t+1)-6u(t-2)+6e^{-0.5(t-2)}u(t-2)$$

At $t = 2$, $y(t) = u(5) + 6u(2) - 6e^{-0.5\times 2}u(2) - u(3) - 6u(0) + 6e^{-0.5\times 0}u(0)$

$$y(t) = 1 + 6 - 6e^{-1} - 1 - 0 + 0$$

$$y(t) = 6(1 - e^{-1}) = 3.79$$ **Ans.**

Ans. 3.6 to 4.0

Q.14 For linear time invariant systems, that are Bounded Input Bounded Output stable, which one of the following statements is TRUE?

(A) The impulse response will be integrable, but may not be absolutely integrable.

(B) The unit impulse response will have finite support.

(C) The unit step response will be absolutely integrable.

(D) The unit step response will be bounded.

Sol. Method 1 :

(1) BIBO stable $h(t) = e^{-at}\,u(t)$

Step response, $y(t) = e^{-at}u(t) \otimes u(t)$

$$Y(s) = \frac{1}{s+a}.\frac{1}{s} \Rightarrow y(t) = \frac{1}{a}[1 - e^{at}]u(t)$$

$y(t)$ is absolutely integrable.

(2) BIBO stable $h(t) = \delta(t)$

Step response, $y(t) = \delta(t) \otimes u(t) = u(t)$

$y(t)$ is not absolutely integrable.

Option (C) is wrong

Option (A) is wrong

The impulse response has finite support means finite impulse response.

FIR filter is a filter whose impulse response is of finite duration, because it settles to zero in finite time.

Option (B) is wrong.

Option (D) is correct as $u(t)$ is bounded.

Method 2 :

Let $h(t) = e^{-at}u(t)$

Option (A) : A system is to be absolutely integrable if,

$$\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$$

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{-\infty}^{\infty} e^{-at}u(t)\,dt = \int_{0}^{\infty} e^{-at}\,dt$$

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \left[\frac{e^{-at}}{-a}\right]_0^{\infty} = \frac{0-1}{-a} = \frac{1}{a} < \infty$$

$\therefore$ System is absolutely integrable.

Hence, option (A) is wrong.

Option (B) : Finite support means finite duration signal

$$h(t) = e^{-at}u(t)$$

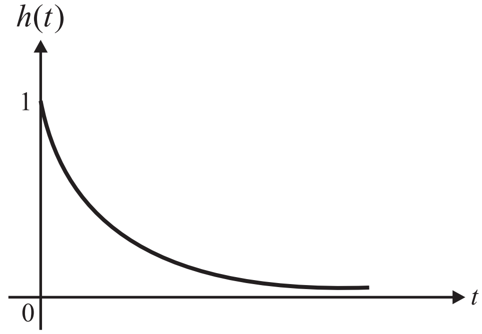


Fig : Impulse response

Amplitude tends to zero as time tends to infinity.

$\therefore$ It is an infinite duration signal and not a finite support signal.

Hence, option (B) is also wrong.

Option (C) : $h(t) = e^{-at}u(t)$

$$H(s) = \frac{1}{s+a}$$

For unit step response,

$$x(t) = u(t) \;\Rightarrow\; X(s) = \frac{1}{s}$$

Output is,

$$Y(s) = X(s) \cdot H(s)$$

$$Y(s) = \frac{1}{s+a} \cdot \frac{1}{s}$$

$$Y(s) = \frac{1}{as} - \frac{1}{a(s+a)}$$

Taking inverse Laplace transform,

$$y(t) = \frac{1}{a}u(t) - \frac{1}{a}e^{-at}u(t)$$

$$y(t) = \frac{1}{a}(1 - e^{-at})u(t)$$

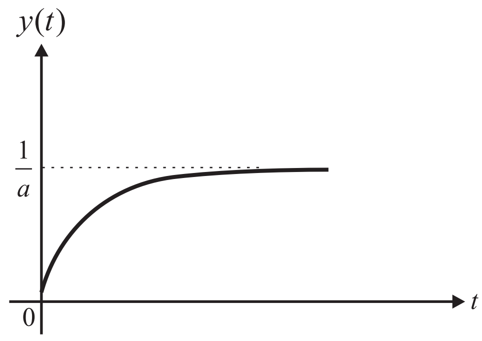


Fig : Unit step response

A signal is said to be absolutely integrable if

$$\int_{-\infty}^{\infty} |y(t)|\, dt < \infty$$

i.e. area of $|y(t)|$ must be less than infinity.

But from figure, area of $y(t)$ is infinite.

Hence, option (C) is wrong.

Option (D) : From figure of unit step response, we see that the step response is a bounded response. Hence, option (D) is correct.

Ans. (D)

Q.15 Consider the function $g(t) = e^{-t}\sin(2\pi t)u(t)$ where $u(t)$ is the unit step function. The area under $g(t)$ is ________.

Sol. Method 1 : Area $= \int_{-\infty}^{\infty} g(t)\, dt = \int_{-\infty}^{\infty} e^{-t}\sin 2\pi t . u(t)\, dt$

$$A = \int_{-\infty}^{\infty} e^{-t}\sin 2\pi t\, dt$$

$$\int e^{-at}\sin bt\, dt = \frac{-e^{-at}}{a^2+b^2}[a\sin bt + b\cos bt]$$

$$b = 2\pi \quad a = 1$$

$$A = \left[\frac{-e^{-t}}{1+4\pi^2}\{\sin 2\pi t + 2\pi\cos 2\pi t\}\right]_0^{\infty}$$

$$= -\left[0 - \frac{2\pi}{1+4\pi^2}\right] = \frac{2\pi}{1+4\pi^2} = 0.155$$

Method 2 : Using LT, $X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\, dt$

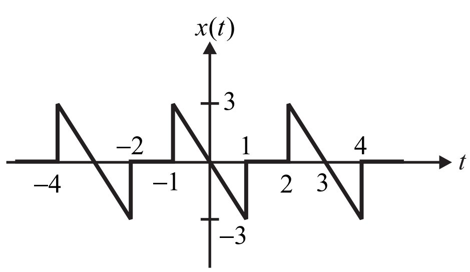
$$X(0) = \int_{-\infty}^{\infty} x(t)\, dt = \text{Area}$$

$$X(s) = \frac{2\pi}{(s+1)^2 + (2\pi)^2}$$

$$X(0) = \frac{2\pi}{1+4\pi^2} = 0.155$$

Ans. 0.155

Q.16 The waveform of a periodic signal $x(t)$ is shown in the figure.



A signal $g(t)$ is defined by $g(t) = x\left(\frac{t-1}{2}\right)$. The average power of $g(t)$ is ________.

Sol.
$$x(t)=\begin{cases}0 & -2<t<-1\\ -3t & -1<t<1\end{cases}$$

$$T_0=3 \qquad (-2 \text{ to } +1)$$

$$P_x=\frac{1}{3}\int_{-2}^{+1}|x(t)|^2\,dt=\frac{1}{3}\int_{-1}^{1}(-3t)^2\,dt$$

$$=\left[t^3\right]_{-1}^{1}=2$$

$$g(t)=x\left(\frac{t-1}{2}\right)=\begin{cases}0 & -2<\frac{t-1}{2}<-1\\ -3\left(\frac{t-1}{2}\right) & -1<\frac{t-1}{2}<1\end{cases}$$

$$g(t)=\begin{cases}0 & -3<t<-1\\ -1.5(t-1) & -1<t<3\end{cases}$$

$$T_0=6 \qquad (-3 \text{ to } +3)$$

$$P_g=\frac{1}{6}\int_{-3}^{3}|g(t)|^2\,dt=\frac{1}{6}\int_{-1}^{3}[-1.5(t-1)]^2\,dt$$

$$=\frac{1.5^2}{6}\left[\frac{(t-1)^3}{3}\right]_{-1}^{3}=\frac{1}{8}\left[(t-1)^3\right]_{-1}^{3}$$

$$=\frac{1}{8}\left[2^3-(-2)^3\right]=2$$

$\therefore \qquad P_g=P_x$

Average power of periodic signal :

(i) Time scaling, Time shifting, Time inversion, Amplitude inversion

$P=$ same

(ii) Amplitude scaling

$P'=(Amp)^2\times P$.

Ans. (2)

Q.17 The response of the system $G(s)=\dfrac{s-2}{(s+1)(s+3)}$ to the unit step input $u(t)$ is $y(t)$.

The value of $\dfrac{dy}{dt}$ at $t=0^+$ is ________.

Sol. Method 1 :

Given : $G(s)=\dfrac{s-2}{(s+1)(s+3)}=\dfrac{Y(s)}{U(s)}$

Where $U(s)=\dfrac{1}{s}=$ Laplace of $u(t)$

$$Y(s)=G(s)\times\frac{1}{s}=\frac{s-2}{s(s+1)(s+3)}$$

Apply initial value theorem, we get

$$y(0) = \lim_{s\to\infty} sY(s)$$

$$y(0) = \lim_{s\to\infty} \frac{s(s-2)}{(s+1)(s+3)} = \frac{\left(1-\frac{2}{s}\right)}{s\left(1+\frac{1}{s}\right)\left(1+\frac{3}{s}\right)}$$

$$y(0) = 0$$

$$L\left[\frac{dy}{dt}\right] = sY(s) = \frac{s(s-2)}{s(s+1)(s+3)} = \frac{(s-2)}{(s+1)(s+3)}$$

$$\left.\frac{dy}{dt}\right|_{t=0} = \lim_{s\to\infty} sL\left[\frac{dy}{dt}\right]$$

$$\left.\frac{dy}{dt}\right|_{t=0} = \lim_{s\to\infty} \frac{s(s-2)}{s(s+1)(s+3)} = \frac{\left(1-\frac{2}{s}\right)}{\left(1+\frac{1}{s}\right)\left(1+\frac{3}{s}\right)} = 1$$

Hence, the correct answer is 1.

Method 2 : $Y(s) = \frac{(s-2)}{s(s+1)(s+3)}$

$$Y(s) = \frac{-2}{3-s} + \frac{3}{2(s+1)} - \frac{5}{6(s+3)}$$

Taking inverse Laplace transform,

$$y(t) = \frac{-2}{3} + \frac{3}{2}e^{-t} - \frac{5}{6}e^{-3t}$$

Now, $\frac{dy(t)}{dt} = \frac{-3}{2}e^{-t} + \frac{5}{2}e^{-3t}$

$$\frac{dy}{dt}(t=0) = \frac{-3}{2} + \frac{5}{2} = \frac{2}{2} = 1$$

Hence, the correct answer is 1.

Ans. 1

Q.18 The output of a continuous-time, linear time-invariant system is denoted by $T\{x(t)\}$ where $x(t)$ is the input signal. A signal $z(t)$ is called eigen-signal of the system T, when $T\{z(t)\} = \gamma\, z(t)$, where γ is a complex number, in general, and is called an eigenvalue of T. Suppose the impulse response of the system T is real and even. Which of the following statements is TRUE?

(A) $\cos(t)$ is an eigen-signal but $\sin(t)$ is not

(B) $\cos(t)$ and $\sin(t)$ are both eigen-signals but with different eigenvalues

(C) $\sin(t)$ is an eigen-signal but $\cos(t)$ is not

(D) $\cos(t)$ and $\sin(t)$ are both eigen-signals with identical eigenvalues

Sol. **Given :** Impulse response is real and even

Thus $H(j\omega_0)$ will also be real and even

Therefore $H(j\omega_0) = H(-j\omega_0)$

Now $\cos(t)$ is input i.e. $\dfrac{e^{it}+e^{it}}{2}$ is input

Output will be $\dfrac{H(j1)e^{it}+H(-j1)e^{-it}}{2} = H(j1)\left[\dfrac{e^{it}+e^{-it}}{2}\right]$

$$= H(j1)\times\cos(t)$$

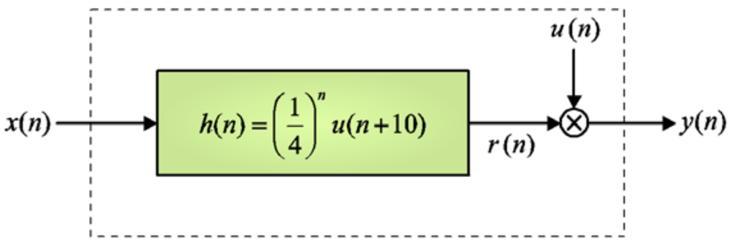
If sin(*t*) is input i.e. $\dfrac{e^{it}-e^{-it}}{2}$ is input

Output will be $\dfrac{H(j1)e^{it}-H(-j1)e^{-it}}{2} = H(j1)\left|\dfrac{e^{it}-e^{-it}}{2}\right|$

So sin(*t*) and cos(*t*) are eigen signal with same eigen values.

Ans. **(D)**

Q.19 Consider the system shown in below figure. The output of an LTI system with an impulse response $h[n]=\left[\dfrac{1}{4}\right]^n u(n+10)$ is multiplied by a unit step function $u(n)$ to yield the output of the overall system.



The overall system is

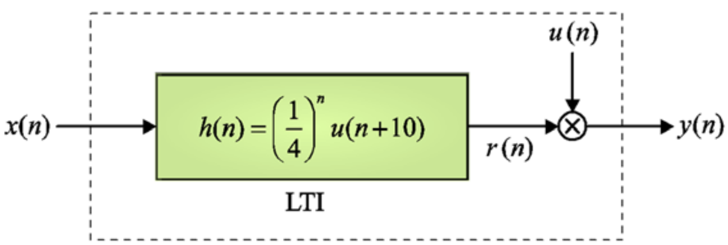
(A) LTI, causal, BIBO stable

(B) Non- LTI, causal, BIBO unstable

(C) Non- LTI, non-causal, BIBO stable

(D) LTI, Non-causal, BIBO stable

Sol.



$$v(n) = x(n)\otimes h(n)$$

$$y(n) = v(n)\cdot u(n)$$

$$y(n) = \left[x(n)\otimes h(n)\right]\cdot u(n)$$

Time invariance :

A system is said to be time-invariant if,

$$y(n-n_0) = T\left[x(n-n_0)\right]$$

$$y(n) = \left[x(n)\otimes h(n)\right]u(n)$$

Introduce n_0 shift in $x(n)$.

$$T\{x(n-n_0)\} = [x(n-n_0) \otimes h(n)]u(n) \qquad \ldots\ldots \text{(A)}$$

Replace all n by $n - n_0$,

$$y(n-n_0) = [x(n-n_0) \otimes h(n-n_0)]u(n-n_0) \qquad \ldots\ldots\ldots \text{(B)}$$

$\because$ (A) $\neq$ (B) $\Rightarrow$ System is time-variant.

$\therefore$ the system is non-LTI.

Causality :

A system is said to be causal if output at any time depends on the present input at that time.

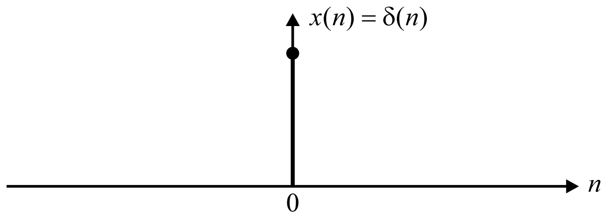
(i) If $x(n) = \delta(n)$

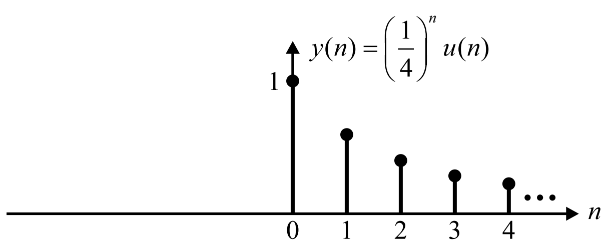
$$v(n) = \left(\frac{1}{4}\right)^n u(n+n_0) \otimes \delta(n)$$

$$v(n) = \left(\frac{1}{4}\right)^n u(n+n_0)$$

$$y(n) = v(n) \cdot u(n) = \left(\frac{1}{4}\right)^n u(n+n_0) \cdot u(n)$$

$$y(n) = \left(\frac{1}{4}\right)^n u(n)$$





From above figure,

For n = 0, output occur when input is applied

Output = f (Present input)

For $n \neq 0$, output occur after input is applied

Output = f (Present input)

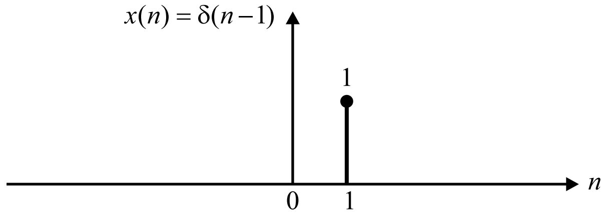
(ii) If $x(n) = \delta(n-1)$

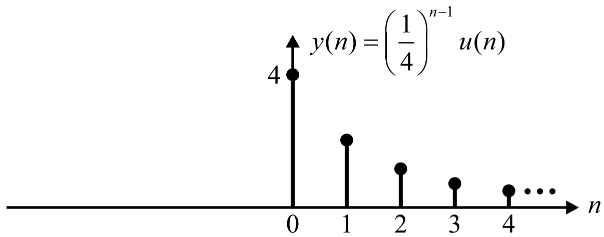
$$v(n) = \left(\frac{1}{4}\right)^n u(n+10) \otimes \delta(n-1)$$

$$v(n) = \left(\frac{1}{4}\right)^{n-1} u(n-1+10) = \left(\frac{1}{4}\right)^{n-1} u(n+9)$$

$$y(n) = v(n) \cdot u(n) = \left(\frac{1}{4}\right)^{n-1} u(n+9) \cdot u(n)$$

$$y(n)=\left(\frac{1}{4}\right)^{n-1}u(n)$$





From above figure,

For n = 0, output occurs before input is applied

i.e. output = f (future input)

For n = 1, output occurs when input is applied

i.e. output = f (present input)

For n > 1, output occurs after input is applied

i.e. output = f (past input)

$\therefore$ It is non-causal because output also depends on the future input.

BIBO :

A system is said to be BIBO stable if bounded input produces bounded output.

$\therefore$ $x(n)$ must be bounded.

$$h(n)=\left(\frac{1}{4}\right)^{n}u(n+10)$$

It is of form $a^n u(n+n_0)$ where $|a|<1$.

$\therefore$ $h(n)$ is stable system.

For a stable system, input and output are stable.

$\therefore$ $v(n)=h(n)\otimes x(n)=$ Stable

$|v(n)|<\infty$

$y(n)=v(n)\cdot u(n)$

$|u(n)|<\infty$

$\therefore$ $y(n)=v(n)\cdot u(n)=$ Stable [$\because$ multiplication of two stable signal is stable signal]

$\therefore$ It is BIBO stable.

Hence, the correct option is (C).

Ans. (C)

Q.20 The transfer function of a system is $\frac{Y(s)}{R(s)} = \frac{s}{s+2}$. The steady state output $y(t)$ is $A\cos(2t+\phi)$ for the input $\cos(2t)$. The values of A and ϕ, respectively are

(A) $\frac{1}{\sqrt{2}}, -45^0$ (B) $\frac{1}{\sqrt{2}}, +45^0$

(C) $\sqrt{2}, -45^0$ (D) $\sqrt{2}, +45^0$

Sol. **Given :** $\frac{Y(s)}{R(s)} = \frac{s}{s+2}$

$r(t) = \text{input} = \cos(2t)$

For $y(t)$ putting $\omega = 2$

i.e. $s = j\omega = 2j$

$$Y(s)\big|_{s=2j} = \frac{2j}{2+2j} R(s)$$

$$y(t) = \frac{2j}{2+2j}\cos 2t$$

$$\Rightarrow \quad y(t) = \frac{1}{\sqrt{2}}\cos(2t+45^0) \quad \text{.... (i)}$$

$$y(t) = A\cos(2t+\phi) \quad \text{.... (ii) (given)}$$

Comparing (i) and (ii) we get,

$$A = \frac{1}{\sqrt{2}}, \quad \phi = 45^0$$ **Ans.**

Ans. **(B)**

Q.21 Let $x(t) = \alpha\beta\, s(t) + \beta s(-t)$ with $s(t) = e^{-4t}u(t)$, where $u(t)$ is unit step function. If the bilateral Laplace transform of $x(t)$ is

$$X(s) = \frac{16}{s^2-16} \qquad -4 < \text{Re}\{s\} < 4;$$

Then the value of β is _______.

Sol. $x(t) = \alpha\beta\ p(t) + \beta p(-t)$

Taking Laplace transform,

$$X(s) = \alpha\beta\ P(s) + \beta P(-s)$$

$$X(s) = \alpha\beta.\frac{1}{s+4} + \beta.\frac{1}{-s+4} = \frac{16}{s^2-16}$$

$$\beta\left[\frac{-\alpha s + 4\alpha + s + 4}{16-s^2}\right] = \frac{16}{s^2-16}$$

$$\frac{-\beta[(1-\alpha)s + 4(1+\alpha)]}{s^2-16} = \frac{16}{s^2-16}$$

$\therefore$ $1-\alpha = 0 \quad \Rightarrow \quad \alpha = 1$

$\Rightarrow$ $\dfrac{-\beta.8}{s^2-16}=\dfrac{16}{s^2-16}$

$\therefore$ $\beta=-2$

Ans. (– 2)

Q.22 A CT system is described by the equation,

$$y(t)=\int_{-\infty}^{t/3} x(\lambda)\,d\lambda$$

The system is

1. Linear
2. Time invariant
3. Stable
4. Invertible

Choose the correct option :

(A) 1, 4

(B) Only 1

(C) 1, 3

(D) 1, 2, 3

Sol. $$y(t)=\int_{-\infty}^{t/3} x(\lambda)\,d\lambda$$

Linearity :

It is of the form $y=mx$.

$\therefore$ It is linear.

We can also check linearity by using concept of additivity and homogeneity.

Time-invariance :

A system is said to be time-invariant if

$$y(t-t_0)=\text{T}[x(t-t_0)]$$

$$y(t)=\int_{-\infty}^{t/3} x(\lambda)\,d\lambda$$

Let us write the above equation as

$$y(t)=\int x\left(\frac{t}{3}\right)dt \qquad \text{(Ignoring the limits)}$$

Direct change in input and output,

$$y(t-t_0)=\int x\left(\frac{t-t_0}{3}\right)dt \qquad \text{....(A)}$$

Direct change in system,

$$y(t-t_0)=\int x\left(\frac{t}{3}-t_0\right)dt \qquad \text{....(B)}$$

$\because$ (A) $\neq$ (B)

$\therefore$ System is time-variant.

Stability :

A system is said to be stable if bounded input produces bounded output.

Let $x(t)=u(t)=\text{Bounded input}$

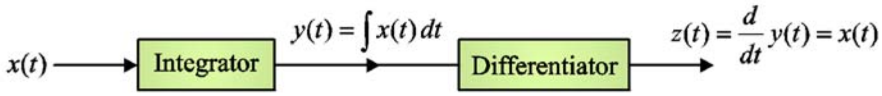
$$y(t)=\int_{-\infty}^{t/3} x(\lambda)\,d\lambda=\int_{-\infty}^{t/3} u(\lambda)\,d\lambda=r(\lambda)=r(t) \qquad \text{(Ignoring the limits)}$$

$$y(t)=r(t)=t\cdot u(t)=\text{Unbounded}$$

$\therefore$ System is unstable.

Invertibility :

A system is said to be invertible if its inverse exists.



So, the system is invertible.

Hence, the correct option is (A).

Ans. (A)

Q.23 The even and odd parts of the signal $g(t)=t(2-t)(1+4t)$ are

(A) $g_e(t)=2-4t^2,\ g_0(t)=7t^3$

(B) $g_e(t)=7t^2,\ g_0(t)=t(2-4t^2)$

(C) $g_e(t)=0,\ g_0(t)=t(2-t)(1+4t)$

(D) $g_e(t)=t(2-t)(1+4t),\ g_0(t)=0$

Sol.

$$g(t)=t(2-t)(1+4t)$$

$$g(t)=t(2+8t-t+4t^2)=2t+7t^2-4t^3$$

Even part of $g(t)=g_e(t)=\dfrac{g(t)+g(-t)}{2}$

$$=\frac{(2t+7t^2-4t^3)+(-2t+7t^2+4t^3)}{2}$$

$$g_e(t)=7t^2$$

Odd part of $g(t)=g_0(t)=\dfrac{g(t)-g(-t)}{2}$

$$=\frac{(2t+7t^2-4t^3)-(-2t+7t^2+4t^3)}{2}$$

$$g_0(t)=2t-4t^3=t(2-4t^2)$$

Hence correct option is (B)

Ans. (B)

Q.24 The value of integral $\displaystyle\int_{\frac{-1}{20}}^{\frac{1}{20}}[4\cos(10\pi t)+8\sin(5\pi t)]dt$ is

(A) $\dfrac{8}{10\pi}$

(B) $\dfrac{4}{10\pi}$

(C) $\dfrac{2}{10\pi}$

(D) $\dfrac{1}{10\pi}$

Sol. Method 1 :

$$I=\int_{-1/20}^{1/20}\left[4\cos(10\pi t)+8\sin(5\pi t)\right]dt$$

$$=4\int_{-1/20}^{1/20}\cos(10\pi t)dt+8\int_{-1/20}^{1/20}\sin(5\pi t)dt$$

$$=\frac{4}{10\pi}[\sin(10\pi t)]_{-1/20}^{1/20}+\frac{(-8)}{5\pi}[\cos(5\pi t)]_{-1/20}^{1/20}$$

$$=\frac{4}{10\pi}\left[\sin\left(\frac{\pi}{2}\right)-\sin\left(\frac{-\pi}{2}\right)\right]+\frac{(-8)}{5\pi}\left[\cos\left(\frac{\pi}{4}\right)-\cos\left(\frac{-\pi}{4}\right)\right]_{-1/20}^{1/20}$$

$[\sin(-\theta)=-\sin\theta,\ \cos(-\theta)=\cos\theta]$

$$=\frac{4}{10\pi}(1+1)$$

$$=\frac{8}{10\pi}$$

Hence correct option is (A)

Method 2 :

$$I=\int_{-1/20}^{1/20}\Big[\underbrace{4\cos(10\pi t)}_{\text{even}}+\underbrace{8\sin(5\pi t)}_{\text{odd}}\Big]dt$$

$\because$ $$\int_{-a}^{a}\text{even}\,dt=2\int_{0}^{a}\text{even}\,dt$$

$$\int_{-a}^{a}\text{odd}\,dt=0$$

$$I=2\int_{0}^{\frac{1}{20}}4\cos(10\pi t)\,dt+0=8\left[\frac{\sin 10\pi t}{10\pi}\right]_{0}^{\frac{1}{20}}=\frac{8}{10\pi}$$

Ans. (A)

Q.25 The power of odd part of the signal $g(t)=20\cos\left(40\pi t-\frac{\pi}{4}\right)$ is

(A) 0 (B) 25

(C) 50 (D) 100

Sol. $$g(t)=20\cos\left(40\pi t-\frac{\pi}{4}\right)$$

Odd part of $g(t)=g_0(t)=\dfrac{g(t)-g(-t)}{2}$

$$=\frac{20\cos\left(40\pi t-\frac{\pi}{4}\right)-20\cos\left(-40\pi t-\frac{\pi}{4}\right)}{2}$$

$$=\frac{20\left[\cos\left(40\pi t-\frac{\pi}{4}\right)-\cos\left(-\left(40\pi t+\frac{\pi}{4}\right)\right)\right]}{2}$$

$[\because\ \cos(-\theta)=-\cos\theta, \cos(A-B)-\cos(A+B)=2\sin A\sin B]$

$$=10\left[\cos\left(40\pi t-\frac{\pi}{4}\right)-\cos\left(40\pi+\frac{\pi}{4}\right)\right]$$

$$=20\sin(40\pi t)\sin\left(\frac{\pi}{4}\right)$$

$$=10\sqrt{2}\ \sin(40\pi t)$$

$$g_0(t) = 10\sqrt{2}\sin(40\pi t)$$

$$\left[\begin{array}{l} \text{power of } A\sin(\omega t \pm \theta) = \dfrac{A^2}{2} \\ \text{power of } A\cos(\omega t \pm \theta) = \dfrac{A^2}{2} \end{array}\right]$$

Power of $g_0(t) = \dfrac{(10\sqrt{2})^2}{2} = 100\text{W}$

Hence correct option is (D)

Ans. (D)

Q.26 A moving average function is given by $y(t) = \frac{1}{T}\int_{t-T}^{t} u(\tau)\,d\tau$. If the input u is a sinusoidal signal of frequency $\frac{1}{2T}$ Hz, then in steady state, the output y will lag u (in degree) by ______.

Sol. $u(\tau) = \sin\omega t$

$$y(t) = \frac{1}{T}\int_{t-T}^{t} \sin(\omega\tau)d\tau = \left.\frac{\cos\omega\tau}{\omega T}\right|_{t-T}^{t}$$

$$y(t) = \frac{1}{\pi}[\cos\omega(t-T) - \cos\omega t]$$

$$\omega T = 2\pi f.T = 2\pi.\frac{1}{2T}.T = \pi$$

$$y(t) = \frac{1}{\pi}[\cos(\omega t - \pi) - \cos\omega t]$$

$$g(t) = \frac{-2}{\pi}\cos\omega t = \frac{2}{\pi}\sin(\omega t - 90^0)$$

$\therefore$ $y(t)$ lags $u(t)$ by 90^0

Ans. (90)

Q.27 The Laplace transform of $f(t) = 2\sqrt{\frac{t}{\pi}}$ is $s^{-\frac{3}{2}}$. The Laplace transform of $g(t) = \sqrt{\frac{1}{\pi t}}$ is

(A) $\dfrac{3s^{-\frac{5}{2}}}{2}$

(B) $s^{-\frac{1}{2}}$

(C) $s^{\frac{1}{2}}$

(D) $s^{\frac{3}{2}}$

Sol. Method 1 :

$$g(t) = \frac{1}{2t}f(t)$$

$$\therefore \quad G(s) = \frac{1}{2}\int_s^{\infty} F(s)\,ds = \frac{1}{2}\int_s^{\infty} s^{-3/2}ds = \frac{1}{2}\left[\frac{s^{\frac{-1}{2}}}{\frac{-1}{2}}\right]_s^{\infty}$$

$$= s^{\frac{-1}{2}}$$

or $$g(t) = \frac{d}{dt} f(t)$$

$\therefore$ $$G(s) = sF(s) = s.s^{-3/2} = s^{-1/2}$$

Method 2 :

$$f(t) = 2\sqrt{\frac{t}{\pi}}$$

$$g(t) = \sqrt{\frac{1}{\pi t}}$$

$$g(t) = \frac{1}{2t} f(t)$$

$$f(t) = 2\sqrt{\frac{t}{\pi}} \xleftrightarrow{\text{L.T.}} F(s) = s^{\frac{-3}{2}}$$

$$g(t) = \frac{f(t)}{2t} \xleftrightarrow{L.T.} \frac{1}{2}\int_s^\infty F(s)\,ds = G(s)$$

$\therefore$ $$G(s) = \frac{1}{2}\left[\frac{s^{\frac{-1}{2}}}{\frac{-1}{2}}\right]_s^\infty$$

$$G(s) = \frac{1}{2}\left[\frac{\infty^{\frac{-1}{2}} - s^{\frac{-1}{2}}}{\frac{-1}{2}}\right]$$

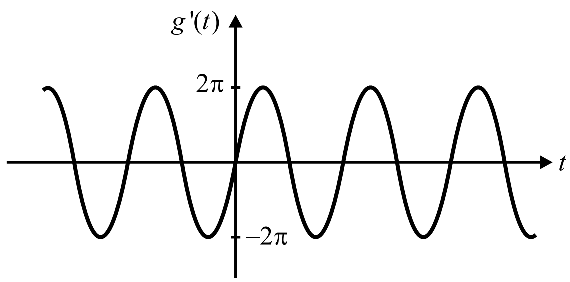
$$G(s) = s^{\frac{-1}{2}} \qquad [\because \ \infty^{-1} = 0]$$

Hence, the correct option is (B).

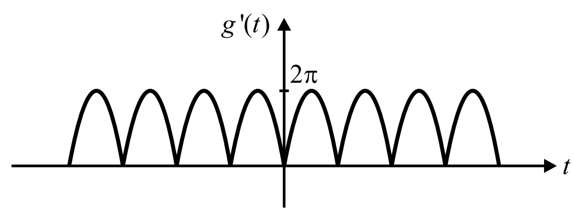
Ans. (B)

Q.28 The derivative is of function $g(t) = |\cos(2\pi t)|$ is

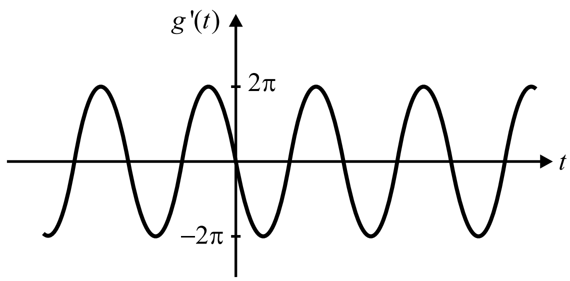
(A)



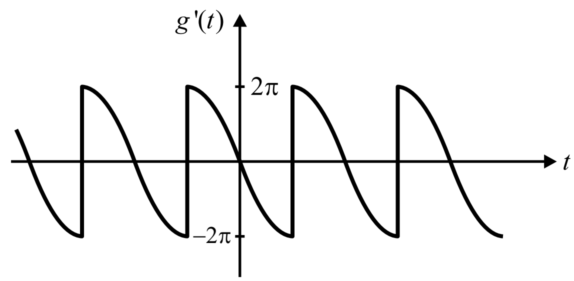
(B)



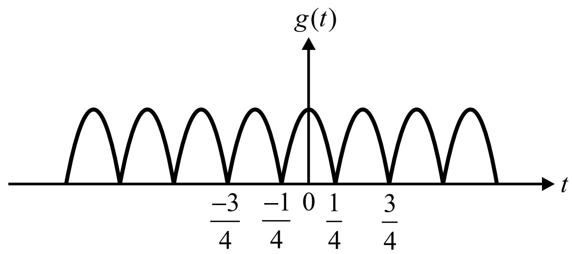
(C)



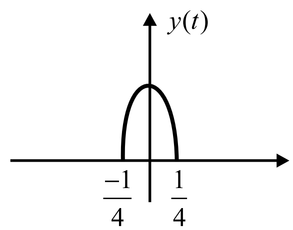
(D)



Sol. $g(t)=|\cos(2\pi t)|$



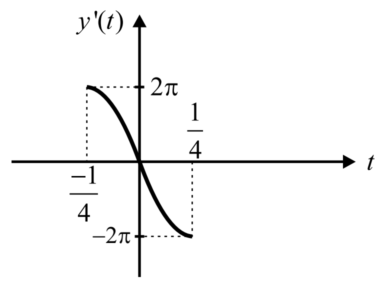
For calculating $g'(t)$, we should calculate only derivative of $y(t)$

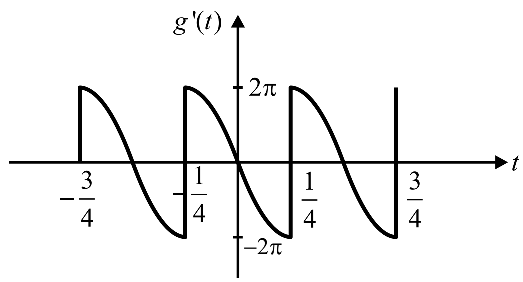


Because this wave form is repeating continuously

$$y(t)=\cos 2\pi t\,,\ -\frac{1}{4}\le t\le\frac{1}{4}$$

$$\frac{dy(t)}{dt}=-2\pi\sin 2\pi t\,,\ -\frac{1}{4}\le t\le\frac{1}{4}$$





Ans. (D)

Q.29 A stable, causal system has a rational transfer function $H(s)$. The system satisfies the following conditions.

(i) The impulse response $h(t)$ is real valued.

(ii) $H(s)$ has exactly two zeros, one of which is at $s=1+j$.

(iii) The signal $\frac{d^2}{dt^2}h(t)+3\frac{d}{dt}h(t)+2h(t)$ contains an impulse and doublet of unknown strengths and a unit amplitude step.

The impulse response $h(t)$ is

(A) $\frac{1}{2}u(t)-5e^{-t}u(t)+5e^{-2t}u(t)$ (B) $u(t)-\frac{5}{2}e^{-t}u(t)+\frac{5}{2}e^{-2t}u(t)$

(C) $\frac{1}{2}u(t)-\frac{5}{2}e^{-t}u(t)+\frac{5}{2}e^{-2t}u(t)$ (D) $u(t)-5e^{-t}u(t)+5e^{-2t}u(t)$

Sol. Method 1 :

$$\frac{d^2}{dt^2}h(t)+3\frac{d}{dt}h(t)+2h(t)=b\delta'(t)+a\delta(t)+u(t)$$

$$H(s)\left[s^2+3s+2\right]=bs+a+\frac{1}{2}=\frac{bs^2+as+1}{s}$$

$\therefore$ $$H(s)=\frac{bs^2+as+1}{s(s^2+3s+2)}$$

One zero is at $s=1+j$ and $h(t)$ is real valued which implies that zeros occur in conjugate pairs, so the other zero is $1-j$.

$\therefore$ $$\left[s-(1+j)\right]\cdot\left[s-(1-j)\right]=(s-1)^2-j^2=s^2-2s+2$$

$\therefore$ $$H(s)=\frac{\frac{1}{2}s^2-s+1}{s(s+1)(s+2)}$$

$$H(s)=\frac{1/2}{s}-\frac{5/2}{s+1}+\frac{5/2}{s+2}$$

$\therefore$ $$h(t)=\frac{1}{2}u(t)-\frac{5}{2}e^{-t}u(t)+\frac{5}{2}e^{-2t}u(t)$$

Method 2 :

1. $h(t)\longrightarrow$ Real valued

2. $H(s)\longrightarrow$ 2 zero's

 Location of one zero is

 $$s_1=1+j$$

 Complex roots always occurs in conjugate pairs

 $\therefore$ $s_2=s_1{*}=(1+j)^*=(1-j)$

 $\therefore$ Zero's are $s=(1+j),\ (1-j)$

3. Equation becomes,

$$\frac{d^2}{dt^2}h(t)+3\frac{d}{dt}h(t)+2h(t)=a\delta'(t)+b\delta(t)+1u(t) \quad \text{....(i)}$$

 Applying Laplace transform, we get

$$s^2H(s)+3sH(s)+2H(s)=as+b+\frac{1}{s}$$

$$H(s)=\frac{as^2+bs+1}{s(s^2+3s+2)}=\frac{as^2+bs+1}{s(s+1)(s+2)} \quad \text{....(ii)}$$

Numerator gives location of zero's

$$k[s-(1+j)]\cdot[s-(1-j)] = k[(s-1)^2 - j^2]$$

$$k[s-(1+j)]\cdot[s-(1-j)] = k[s^2 - 2s + 2]$$

Where k is a system constant.

Comparing $k[s^2 - 2s + 2]$ with numerator of equation (ii),

$$as^2 + bs + 1 = ks^2 - 2ks + 2k$$

$$2k = 1 \Rightarrow k = \frac{1}{2}$$

$$a = k = \frac{1}{2}$$

$$b = -2k = -2\times\frac{1}{2} = -1$$

$\therefore$ From equation (ii),

$$H(s) = \frac{\frac{1}{2}\left(s^2 - s + 1\right)}{s(s+1)(s+2)}$$

$$H(s) = \frac{\frac{1}{2}}{s} - \frac{\frac{s}{2}}{s+1} + \frac{\frac{5}{2}}{s+2}$$

By inverse Laplace transform,

$$h(t) = \frac{1}{2}u(t) - \frac{5}{2}e^{-t}u(t) + \frac{5}{2}e^{-2t}u(t)$$

Hence, the correct option is (C).

Ans. (C)

Q.30 Suppose the following facts are given about the signal $x(t)$ with Laplace transform $X(s)$.

(i) $x(t)$ is real and even.

(ii) $X(s)$ has 4 poles and no zeros in a finite s-plane

(iii) $X(s)$ has a pole at $s = \frac{1}{2}e^{j\frac{\pi}{4}}$

(iv) $\int_{-\infty}^{\infty} x(t)dt = 4$

Then $X(s)$ along with ROC is

(A) $\frac{4}{16s^4+1}$; $\text{Re}(s) > \frac{1}{2\sqrt{2}}$

(B) $\frac{4}{16s^4+1}$; $\text{Re}(s) > 0$

(C) $\frac{16}{16s^4+1}$; $\text{Re}(s) > \frac{1}{2\sqrt{2}}$

(D) $\frac{16}{16s^4+1}$; $\text{Re}(s) > 0$

Sol. 1. $x(t) =$ Real and even.

$\therefore$ $x(t) = x(-t)$

Also $X(s) = X(-s)$

2. $X(s)$ has 4 poles and no zero's.

3. Location of one pole,

$$s = \frac{1}{2}e^{j\frac{\pi}{4}} = \frac{1}{2\sqrt{2}}(1+j)$$

4. $$\int_{-\infty}^{\infty} x(t)\,dt = 4$$

$$X(s) = \frac{k}{(s-P_1)(s-P_2)(s-P_3)(s-P_4)}$$

$$X(-s) = \frac{k}{(-s-P_1)(-s-P_2)(-s-P_3)(-s-P_4)}$$

$$P_1 = \frac{1+j}{2\sqrt{2}}$$

We know that complex poles always occurs in pair i.e. complex poles are conjugate pairs.

$\therefore$ $$P_2 = P_1^* = \frac{1-j}{2\sqrt{2}}$$

$\because$ $$X(s) = X(-s)$$

$$\frac{k}{\left(s-\frac{1+j}{2\sqrt{2}}\right)\left(s-\frac{1-j}{2\sqrt{2}}\right)(s-P_3)(s-P_4)} = \frac{k}{\left(s+\frac{1+j}{2\sqrt{2}}\right)\left(s+\frac{1+j}{2\sqrt{2}}\right)(s+P_3)(s+P_4)}$$

(A) (B) (C) (D) (I) (II) (III) (IV)

Equating (A) = (III), (B) = (IV) [or equate (C) = (I) and (D) = (II)], we get

$$P_3 = -\left(\frac{1+j}{2\sqrt{2}}\right),\ P_4 = -\left(\frac{1-j}{2\sqrt{2}}\right)$$

$\therefore$ $$X(s) = \frac{k}{\left(s-\frac{1+j}{2\sqrt{2}}\right)\left(s-\frac{1-j}{2\sqrt{2}}\right)\left(s+\frac{1+j}{2\sqrt{2}}\right)\left(s+\frac{1-j}{2\sqrt{2}}\right)}$$

$$X(s) = \frac{k}{\left[\left(s-\frac{1}{2\sqrt{2}}\right)^2-\left(\frac{j}{2\sqrt{2}}\right)^2\right]\left[\left(s+\frac{1}{2\sqrt{2}}\right)^2-\left(\frac{j}{2\sqrt{2}}\right)^2\right]}$$

$$X(s) = \frac{k}{\left[s^2-\frac{s}{\sqrt{2}}+2\times\left(\frac{1}{2\sqrt{2}}\right)^2\right]\left[s^2+\frac{s}{\sqrt{2}}+2\times\left(\frac{1}{2\sqrt{2}}\right)^2\right]}$$

$$X(s) = \frac{k}{\left(s^2+\frac{1}{4}-\frac{s}{\sqrt{2}}\right)\left(s^2+\frac{1}{4}+\frac{s}{\sqrt{2}}\right)}$$

$$X(s) = \frac{k}{\left(s^2+\frac{1}{4}\right)^2-\left(\frac{s}{\sqrt{2}}\right)^2} = \frac{k}{s^4+\frac{1}{16}}$$

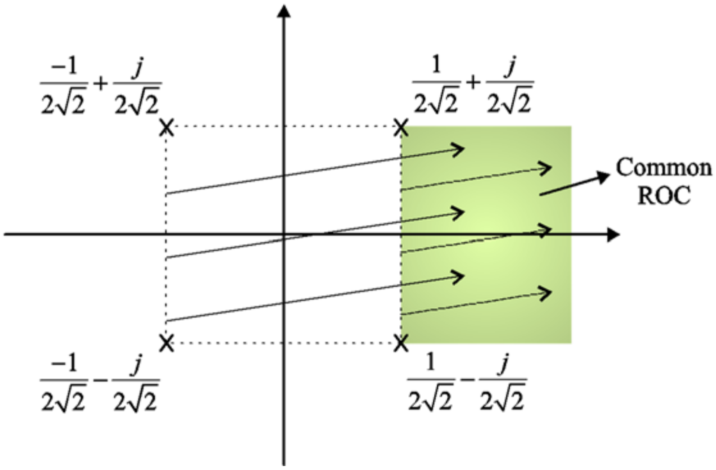
From definition of Laplace transform,

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

Put $s=0$, $$X(0) = \int_{-\infty}^{\infty} x(t)\,dt = \text{Area of } x(t)$$

$\Rightarrow \qquad \dfrac{k}{0+\dfrac{1}{16}} = 4 \Rightarrow k = \dfrac{1}{4}$

$\therefore \qquad X(s) = \dfrac{\dfrac{1}{4}}{s^4 + \dfrac{1}{16}} = \dfrac{4}{16s^4+1}$



$\therefore \quad$ ROC is $\text{Re}(s) > \dfrac{1}{2\sqrt{2}}$

$\therefore \quad X(s) = \dfrac{4}{16s^4+1}, \;\; \text{ROC}: \text{Re}(s) > \dfrac{1}{2\sqrt{2}}$

Hence, the correct option is (A).

Ans. (A)

Q.31 The Laplace transform of $\dfrac{\cos t}{t}u(t)$ and $\dfrac{\sin^2 at}{t}u(t)$ are

(A) $\dfrac{1}{2}\ln(s^2+1), \dfrac{1}{4}\ln\left(1+\dfrac{4a^2}{s^2}\right)$

(B) $-\dfrac{1}{2}\ln(s^2+1), \dfrac{1}{4}\ln\left(1+\dfrac{4a^2}{s^2}\right)$

(C) $\ln\left(\dfrac{s}{s^2+1}\right), \dfrac{1}{2}\ln\left(\dfrac{\sqrt{s^2+4a^2}}{s}\right)$

(D) None of the above

Sol. **(i)** $\dfrac{\cos t}{t}u(t) = \displaystyle\int_S^\infty \dfrac{s}{s^2+1}ds = \left[\dfrac{1}{2}\log_e(s^2+1)\right]_S^\infty = \infty$ [Integration property : $\dfrac{\cos t}{t}\Big|_{t=0} = \dfrac{1}{0} = \infty$]

(ii) $\dfrac{\sin^2 at}{t}u(t) = \left[\dfrac{1-\cos 2at}{2t}\right]u(t) = \dfrac{1}{2}\displaystyle\int_S^\infty\left(\dfrac{1}{s} - \dfrac{s}{s^2+4a^2}\right)ds$

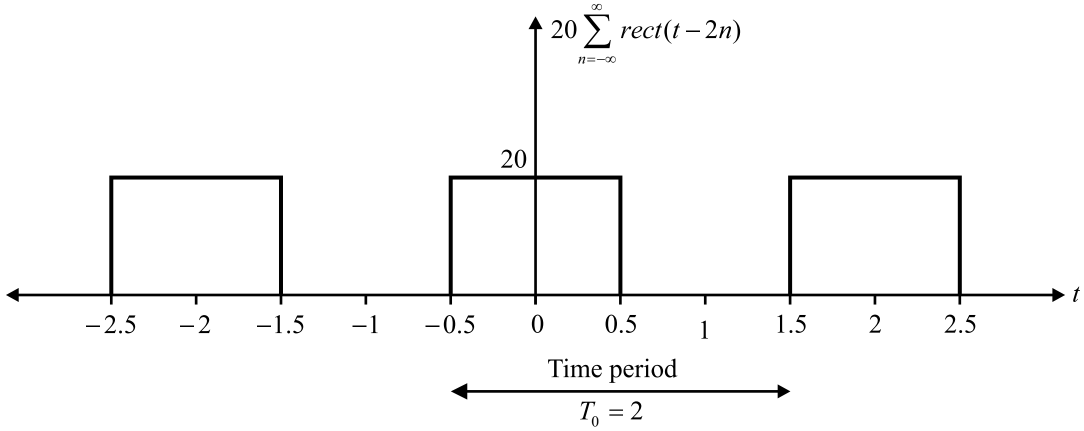
$= \dfrac{1}{2}\left[\ln s - \dfrac{1}{2}\ln(s^2+4a^2)\right]_S^\infty$

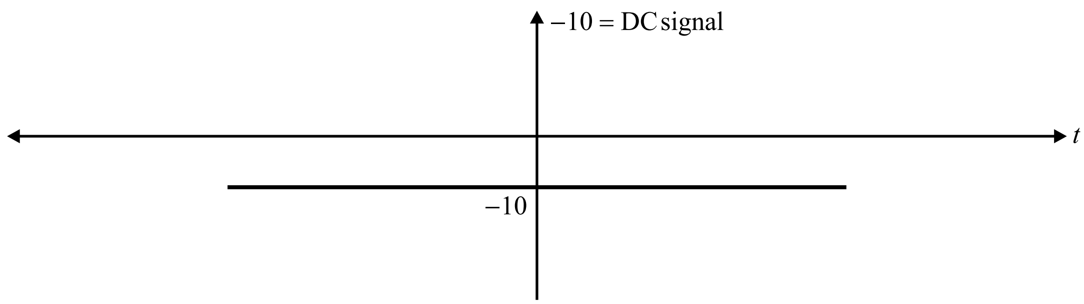
$= \dfrac{1}{2}\left[\ln\left\{\dfrac{s}{\sqrt{s^2+4a^2}}\right\}\right]_S^\infty = \dfrac{1}{2}\ln\left(\dfrac{\sqrt{s^2+4a^2}}{s}\right)$

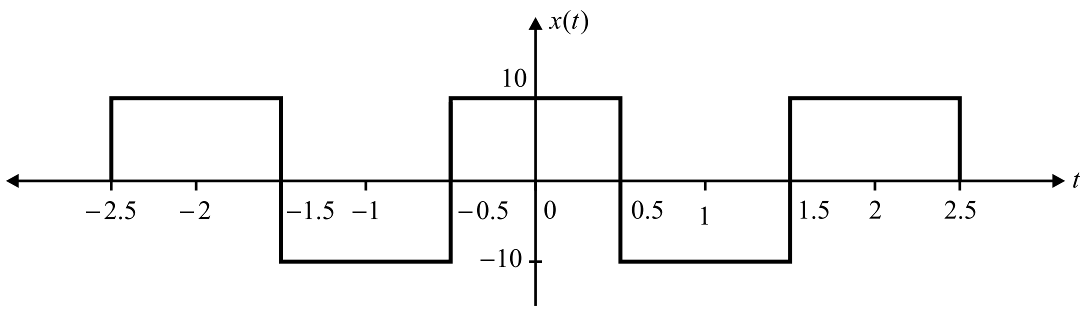
Ans. (D)

Q.32 The power of the signal $x(t) = 20\left[\frac{-1}{2} + \sum_{n=-\infty}^{\infty} rect(t-2n)\right]$ is ______________.

Sol. $$x(t) = 20\left[\frac{-1}{2} + \sum_{n=-\infty}^{\infty} rect(t-2n)\right]$$







The power of periodic signal is given by,

$$P = \frac{1}{2}\int_{T_0} x^2(t)\, dt$$

$$P = \frac{1}{2}\left[\int_{-0.5}^{0.5} 10^2\, dt + \int_{0.5}^{1.5} (-10)^2\, dt\right]$$

$$P = \frac{1}{2}\left\{100[t]_{-0.5}^{0.5} + 100[t]_{0.5}^{1.5}\right\}$$

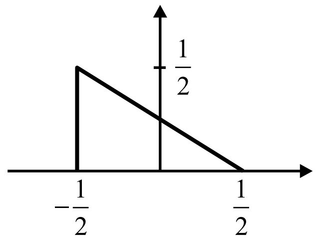
$$P = \frac{1}{2}\{100\times 1 + 100\times 1\}$$

$$P = 100$$ **Ans.**

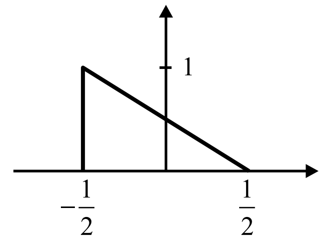
Ans. (100)

Q.33 The waveform of $u\left(t+\frac{1}{2}\right)\cdot r\left(\frac{1}{2}-t\right)$ is

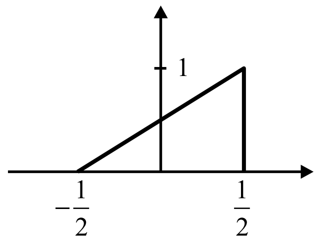
(A)



(B)



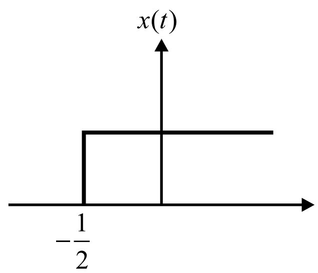
(C)



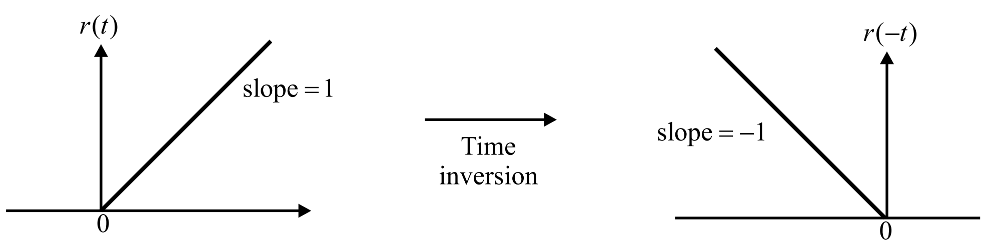
(D) None of the above

Sol. **Wave form of $u\left(t+\frac{1}{2}\right)r\left(\frac{1}{2}-t\right)$:**

(i) Wave form of $u\left(t+\frac{1}{2}\right)=x(t)$

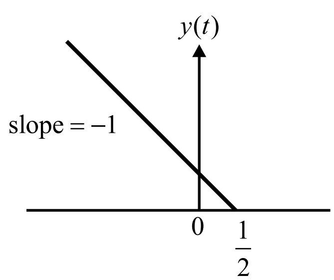


(ii) Wave form of $y(t)=r\left(\frac{1}{2}-t\right)$

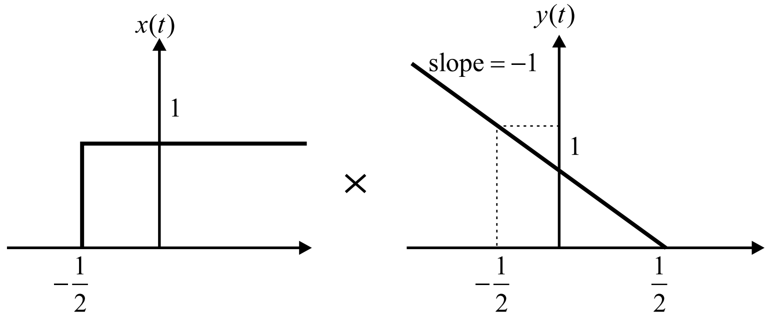


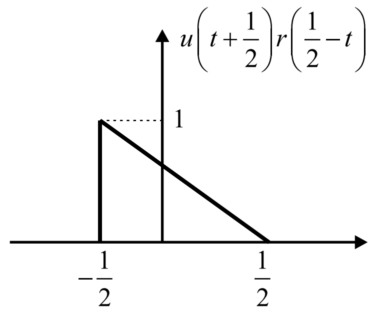
Delay $t\Rightarrow t-\frac{1}{2}$,

$$r\left(-\left(t-\frac{1}{2}\right)\right)=r\left(\frac{1}{2}-t\right)=y(t)$$



Wave form of $u\left(t+\frac{1}{2}\right)r\left(\frac{1}{2}-t\right)$ = multiplication of $x(t)$ and $y(t)$





Hence correct option is (B)

Ans. (B)

Q.34 Consider a causal LTI system characterized by differential equation $\frac{dy(t)}{dt}+\frac{1}{6}y(t)=3x(t)$. The response of the system to the input $x(t)=3e^{-\frac{t}{3}}u(t)$, where $u(t)$ denotes the unit step function, is

(A) $9e^{-\frac{t}{3}}u(t)$ (B) $9e^{-\frac{t}{6}}u(t)$

(C) $9e^{-\frac{t}{3}}u(t)-6e^{-\frac{t}{6}}u(t)$ (D) $54e^{-\frac{t}{6}}u(t)-54e^{-\frac{t}{3}}u(t)$

Sol. Given : The differential equation

$$\frac{dy(t)}{dt}+\frac{1}{6}y(t)=3x(t)$$

Taking Laplace transform we get

$$sY(s)+\frac{1}{6}Y(s)=3\times X(s)$$

$$Y(s)=\frac{3\times X(s)}{\left(s+\frac{1}{6}\right)} \quad \text{.... (i)}$$

And also $x(t)=3e^{-t/3}u(t)$ (given)

Taking Laplace $X(s)=\frac{3}{s+\frac{1}{3}}$ .... (ii)

Putting equation (ii) in equation (i) we get

$$Y(t)=\frac{3}{\left(s+\frac{1}{6}\right)}\times\frac{3}{\left(s+\frac{1}{3}\right)}$$

Applying partial section $=\frac{9}{\left(s+\frac{1}{6}\right)\left(s+\frac{1}{3}\right)}=\frac{A}{s+\frac{1}{6}}+\frac{B}{s+\frac{1}{3}}$

Evaluating A and B, we get

$$A=\left(s+\frac{1}{6}\right)Y(s)\Big|_{s=\frac{-1}{6}}=\frac{9}{\frac{-1}{6}+\frac{1}{3}}=54$$

$$B=\left(s+\frac{1}{3}\right)Y(s)\Big|_{s=\frac{-1}{3}}=\frac{9}{\frac{-1}{3}+\frac{1}{6}}=-54$$

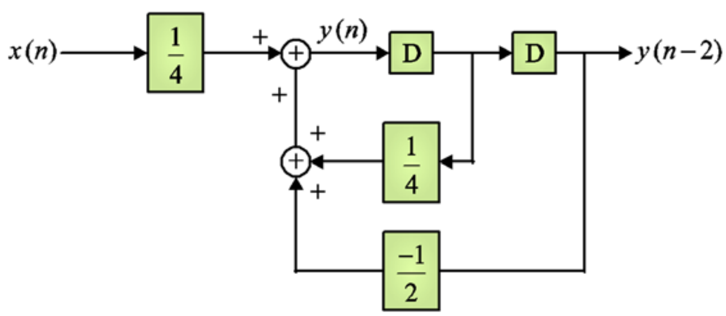
Therefore $Y(s)=\frac{54}{s+\frac{1}{6}}-\frac{54}{s+\frac{1}{3}}$

Now taking inverse Laplace of above, we get

$$y(t)=54e^{-t/6}u(t)-54e^{-t/3}u(t)$$ **Ans.**

Ans. (D)

Q.35 The system shown in block diagram below is



(A) Non-linear, dynamic, causal, time invariant unstable, non-invertible

(B) Linear, dynamic, causal, time invariant, stable, non-invertible

(C) Linear, dynamic, causal, time invariant, stable, invertible

(D) Linear, dynamic, causal, time invariant, unstable, non-invertible

Sol.

$$y[n]=\frac{1}{4}x[n]+\frac{1}{4}y[n-1]-\frac{1}{2}y[n-2]$$

$$x[n]=4y[n]-y[n-1]+2y[n-2]$$

Writing equation from the block diagram,

$$y[n]=\frac{1}{4}x[n]+\frac{1}{4}y[n-1]-\frac{1}{2}y[n-2]$$

$$y[n]-\frac{1}{4}y[n-1]+\frac{1}{2}y[n-2]=\frac{1}{4}x[n]$$ ….(i)

The above equation is a difference equation.

Note : The difference equation is always dynamic and causal.

$$\underbrace{y[n]-\frac{1}{4}y[n-1]+\frac{1}{2}y[n-2]}_{y_1[n]}=\frac{1}{4}x[n]$$

$$y_1[n]=\frac{1}{4}x[n]$$

The equation is of the form

$$y = mx$$

$\therefore$ It is linear.

$\because$ There is no extra "n" term in equation (i).

$\therefore$ It is time-invariant.

A system is said to be invertible if its inverse exists.

Applying Z-transform on both sides of equation (i),

$$Y(z)-\frac{1}{4}z^{-1}Y(z)+\frac{1}{2}z^{-2}Y(z)=\frac{1}{4}X(z)$$

$$Y(z)\left[1-\frac{1}{4}z^{-1}Y(z)+\frac{1}{2}z^{-2}\right]=\frac{1}{4}X(z)$$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{1}{4\left(1-\frac{1}{4}z^{-1}+\frac{1}{2}z^{-2}\right)}$$

$$H(z)=\frac{\frac{z^2}{4}}{z^2-\frac{1}{4}z+\frac{1}{2}}=\frac{\frac{z^2}{4}}{\left(z-\frac{1+j\sqrt{31}}{8}\right)\left(z-\frac{1-j\sqrt{31}}{8}\right)}$$

$$z=\frac{1\pm j\sqrt{31}}{8}$$

$$|z|=\frac{1}{8}\sqrt{1^2+\left(\sqrt{31}\right)^2}=\frac{1}{\sqrt{2}}=0.707$$

$\because$ $|z|<1$

i.e. ROC lies inside unit circle

$\therefore$ System is causal.

Inverse of the system is,

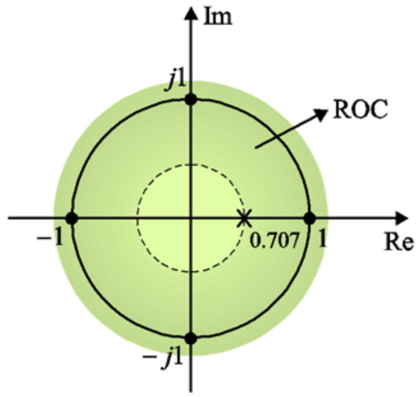
$$H_{inv}(z)=\frac{1}{H(z)}=4\left(1-\frac{1}{4}z^{-1}+\frac{1}{2}z^{-2}\right)$$

Taking inverse z-transform,

$$h_{inv}(n)=4\left[\delta(n)-\frac{1}{4}\delta(n-1)+\frac{1}{2}\delta(n-2)\right]$$

$\because$ $h_{inv}(n)$ exists $\Rightarrow$ System is invertible.

Stability :



∵ The system is causal.

∴ ROC is right side of the outermost pole.

∵ ROC includes unit circle.

∴ System is stable.

Hence, the correct option is (C).

Ans. (C)

❖❖❖❖